MA 351 Fall 2024 (Aaron N. K. Yip)

Penney, Linear Algebra: Ideas and Applications (4th edition)

Homework 11: Due: Thursday, Nov. 21st, in class
EXERCISES p.263: 4.28, 4.29;
EXERCISES p.268: 4.34 (in addition, verify your answer using row reduction).
EXERCISES p.268: 4.36. Do the following for this problem:

1. Solve for x, y, z, using Cramer's Rule.
2. Solve for x, y, z, using row reduction.
3. Find the inverse of the coefficient matrix of the system using the formula for matrix inverse, Theorem 4.15. Using the inverse to solve for x, y, z.

   (Note: it might be easier to replace the right hand side of (4.11) by $\left[\dfrac{(-1)^{i+j}\det(A_{ij})}{\det A}\right]^T$.)

EXERCISES p.280: 5.3, 5.5(acegi), 5.11, 5.12, 5.13, 5.14.

<u>Additional Problem.</u> This problem guides you through the proof of the amazing fact that for *square* matrices A and B, if $AB = I$, then $BA = I$ (and hence A and B are inverse of each other).

START

Let A and B be two $n \times n$ matrices such that $AB = I$.

1. Prove that $AX = Y$ is solvable for any Y (by writing down an explicit formula for X). Verify that your formula actually works.
2. What can you say about the rank of A?
3. What can you say about the rank of A^T? (Think about the meaning of rank in relation to the dimensions of column and row spaces of a matrix.)
4. Prove that the equation $A^T X = Y$ is solvable for any Y.
5. Prove that there is an $n \times n$ matrix C such that $A^T C = I$. (Think about how you would actually find this C, column by column.)
6. Prove that $C^T A = I$.
7. Prove that $C^T = B$ (and hence $BA = I$).

END