Duality – Interpretations

Dual

$$\max \quad f(x) = c^T x$$
$$\text{s.t.} \quad Ax \leq b$$
$$x \geq 0$$

$$\min \quad g(y) = b^T y$$
$$\text{s.t.} \quad A^T y \geq c$$
$$y \geq 0$$

Diet Problem

m Nutrients (eg. Vitamins) N_i : $i = 1, 2, \cdots, m$

n Foods F_j : $j = 1, 2, \cdots, n$

a_{ij} = amount of N_i in 1 unit of F_j

b_i = minimum (monthly) intake of N_i

c_j = price (cost) of F_j

x_j = amount of consumption of F_j

Diet Problem

N_i \ F_j	C_1 1	C_2 2	C_3 3	C_j j	----	C_n n
b_1 1						
b_2 2						
b_i i				a_{ij}		
⋮ ⋮						
b_m m						

Diet Problem

N_i \ F_j	c_1 1	c_2 2	c_3 3	c_j j	----	c_n n
b_1 1						
b_2 2						
b_i i				a_{ij}		
⋮ ⋮						
b_m m						

Diet Problem

cost of C_j

consumption of F_j

minimize cost: $f(x) = \sum_j c_j x_j$

s.t. $\sum_j a_{ij} x_j \geq b_i$ ← min. intake of N_i

$x_j \geq 0$

Diet Problem

N_i \ F_j	C_1 1	C_2 2	C_3 3	C_j j	----	C_n n
b_1 1						
b_2 2						
b_i i				a_{ij}		
⋮ ⋮						
b_m m						

Diet Problem

Replacement of food by pills for each N_i's

N_i = pills $\quad i = 1, 2, \cdots, m$

y_i = unit price of pill for N_i

1 unit of F_j contains a_{ij} unit of N_i

Price/Value of (1 unit) $F_j = \sum_i y_i \cdot a_{ij}$

Maximize profit by selling pills

Diet Problem

Maximize profit by selling pills

$$\max (\text{profit}) \quad \xi(Y) = \sum_i b_i y_i$$

b_i: min. sale of N_i pill

y_i: price of N_i pill

$$\text{s.t.} \quad \sum_i y_i a_{ij} \leq c_j$$

c_j: cost of F_j

a_{ij}: "pill equivalent" of F_j

$$y_i \geq 0$$