

Sensitivity and Parametric Analysis

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$$\text{maximize } 5x_1 + 4x_2 + 3x_3$$

$$\text{subject to } 2x_1 + 3x_2 + x_3 \leq 5$$

$$4x_1 + x_2 + 2x_3 \leq 11$$

$$3x_1 + 4x_2 + 2x_3 \leq 8$$

$$x_1, x_2, x_3 \geq 0 .$$

Sensitivity and Parametric Analysis

Opt.
Dict.

$$\xi = 13 - 3x_2 - 1x_4 - 1x_6$$

$x_3 =$	1	+	x_2	+	3	x_4	-	2	x_6
$x_1 =$	2	-	2	x_2	-	2	x_4	+	x_6
$x_5 =$	1	+	5	x_2	+	2	x_4		

Sensitivity and Parametric Analysis

Opt.
Dict.

$$z_N^* = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = (\bar{B}^{-1}N)^T C_B - C_N, \quad X_N = \begin{pmatrix} x_2 \\ x_4 \\ x_6 \end{pmatrix}$$

$$\xi = 13 \quad \boxed{-3x_2 - 1x_4 - 1x_6}$$

$$X_B = \begin{pmatrix} x_3 \\ x_1 \\ x_5 \end{pmatrix} = \begin{array}{r} \boxed{1} + x_2 + 3x_4 - 2x_6 \\ 2 - 2x_2 - 2x_4 + x_6 \\ 1 + 5x_2 + 2x_4 \end{array}$$

$$X_B^* = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \bar{B}^{-1}b \quad -\bar{B}^{-1}N = \begin{pmatrix} 1 & 3 & -2 \\ -2 & -2 & 1 \\ 5 & 2 & 0 \end{pmatrix}$$

Sensitivity and Parametric Analysis

What if

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Sensitivity and Parametric Analysis

$$X_B^* = B^{-1}b \geq 0 \quad \text{not changed}$$

$$Z_N^* = (B^{-1}N)^T C_B - C_N$$

$$\Delta Z_N^* = (B^{-1}N)^T \Delta C_B - \Delta C_N$$

We need $Z_N^* + \Delta Z_N^* \geq 0$

Sensitivity and Parametric Analysis

What if

$$\text{maximize } 5x_1 + 4x_2 + 3x_3$$

$$\text{subject to } 2x_1 + 3x_2 + x_3 \leq 5 + \Delta b_1$$

$$4x_1 + x_2 + 2x_3 \leq 11 + \Delta b_2$$

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Sensitivity and Parametric Analysis

$$Z_N^* = (B^{-1}N)^T C_B - C_N \geq 0 \text{ not changed}$$

$$X_B^* = B^{-1}b$$

$$\Delta X_B^* = B^{-1} \Delta b$$

$$\vec{\Delta b} = \begin{pmatrix} \Delta b_1 \\ \Delta b_2 \\ \Delta b_3 \end{pmatrix}$$

We need $X_B^* + \Delta X_B^* \geq 0$