# Regression

Given $\{x_1, x_2, \cdots, x_m\}$ $x_i \in \mathbb{R}$

(1) $\min_{x} \frac{1}{m} \sum_{i=1}^{m} (x - x_i)^2 \Longrightarrow x = \frac{x_1 + \cdots + x_m}{m}$

$L^2$-error

(mean)

(2) $\min_{x} \frac{1}{m} \sum_{i=1}^{m} |x - x_i| \Longrightarrow x =$ median

$L^1$-error

(3) $\min_{x} \left( \max_{i} |x - x_i| \right) \Longrightarrow x = \frac{\min x_i + \max x_i}{2}$

$L^\infty$-error

"mid-range"

Given attributes $\{a_1, a_2, \cdots, a_n\}$, $b$

find parameters $\{x_1, x_2, \cdots, x_n\}$ s.t.

$$b = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n$$

$$\left( b = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n + \varepsilon \right)$$

error

Given data points (individuals)

$$b_i, \quad \{a_{ij}\}_{j=1}^{n}, \quad i = 1, \dots, m$$

Find $x_1, x_2, \dots x_n$ s.t.

$$b_i = \sum_{j=1}^{n} a_{ij} x_j, \quad i = 1, \dots, m$$

error

$$b_i = \sum_{j=1}^{n} a_{ij} x_j + \varepsilon_i \quad i = 1, \dots, m$$

$$\underline{b = AX + \varepsilon}$$

# $L^2$-Regression (Least Square)

$$\min_{(x_1,\dots,x_n)} \sum_i \left[ \left( b_i - \sum_j a_{ij} x_j \right)^2 \right]$$

$$\min_x \| b - Ax \|_2^2$$

$Ax = b$ (might not be solvable)

$A^T A \hat{x} = A^T b$ (normal equation, always solvable)

$\hat{x} = (A^T A)^{-1} A^T b$ (if $(A^T A)^{-1}$ exists)

## $L^1$ - regression

$$\min_{(x_1,\dots,x_n)} \sum_i \left| b_i - \sum_j a_{ij} x_j \right|$$

$$\min_{x} \| b - Ax \|_1$$

$$\boxed{\begin{aligned} &\min_{i} \sum_{i=1}^{m} t_i \\ &\text{s.t.} \quad -t_i \le b_i - \sum_j a_{ij} x_j \le t_i \end{aligned}}$$

$|b_i - \sum_j a_{ij} x_j| \le t_i$

## $L^\infty$-Regression

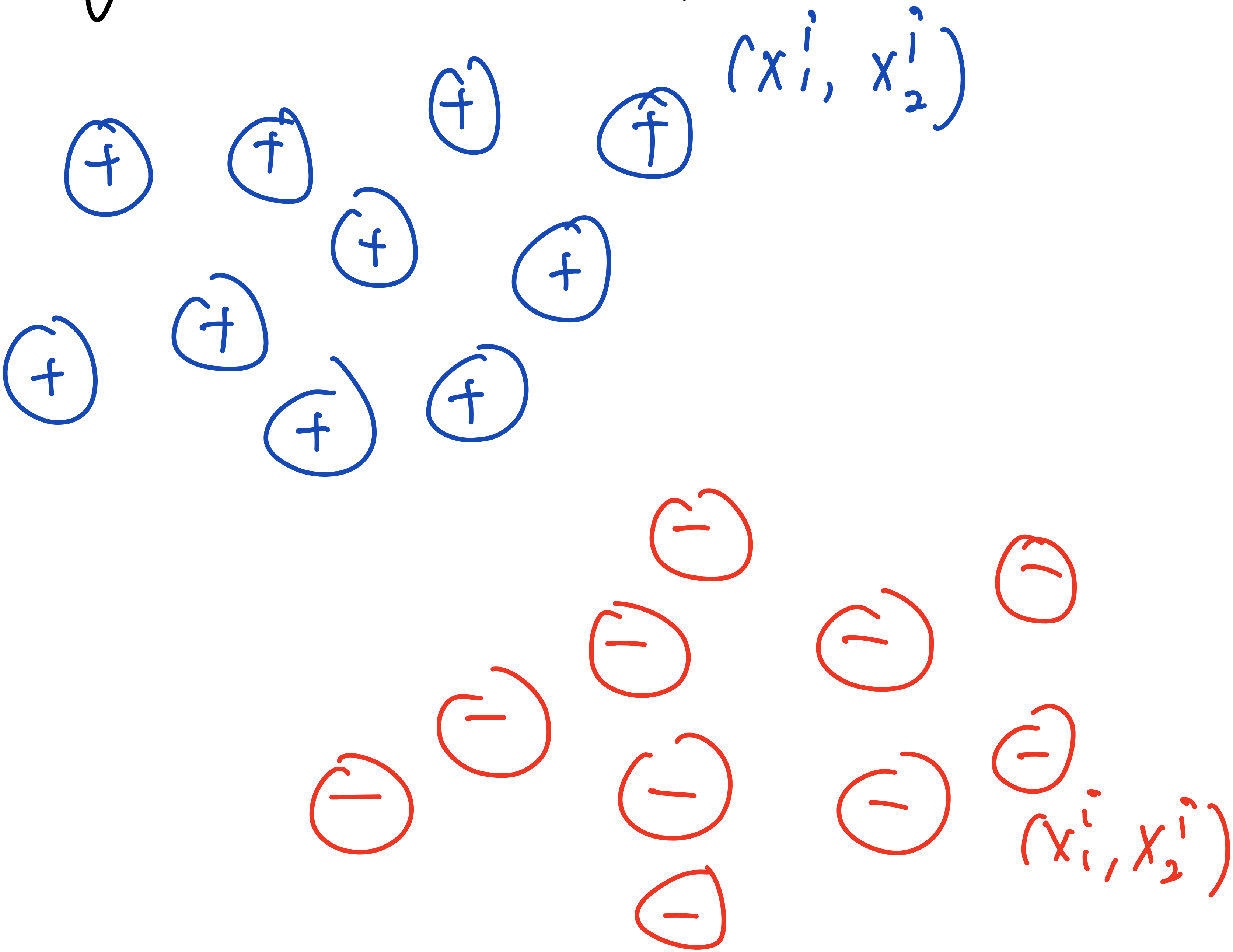
$$\min \left( \max_i \left| b_i - \sum_j a_{ij} x_j \right| \right)$$

$$\min_X \| b - AX \|_\infty$$

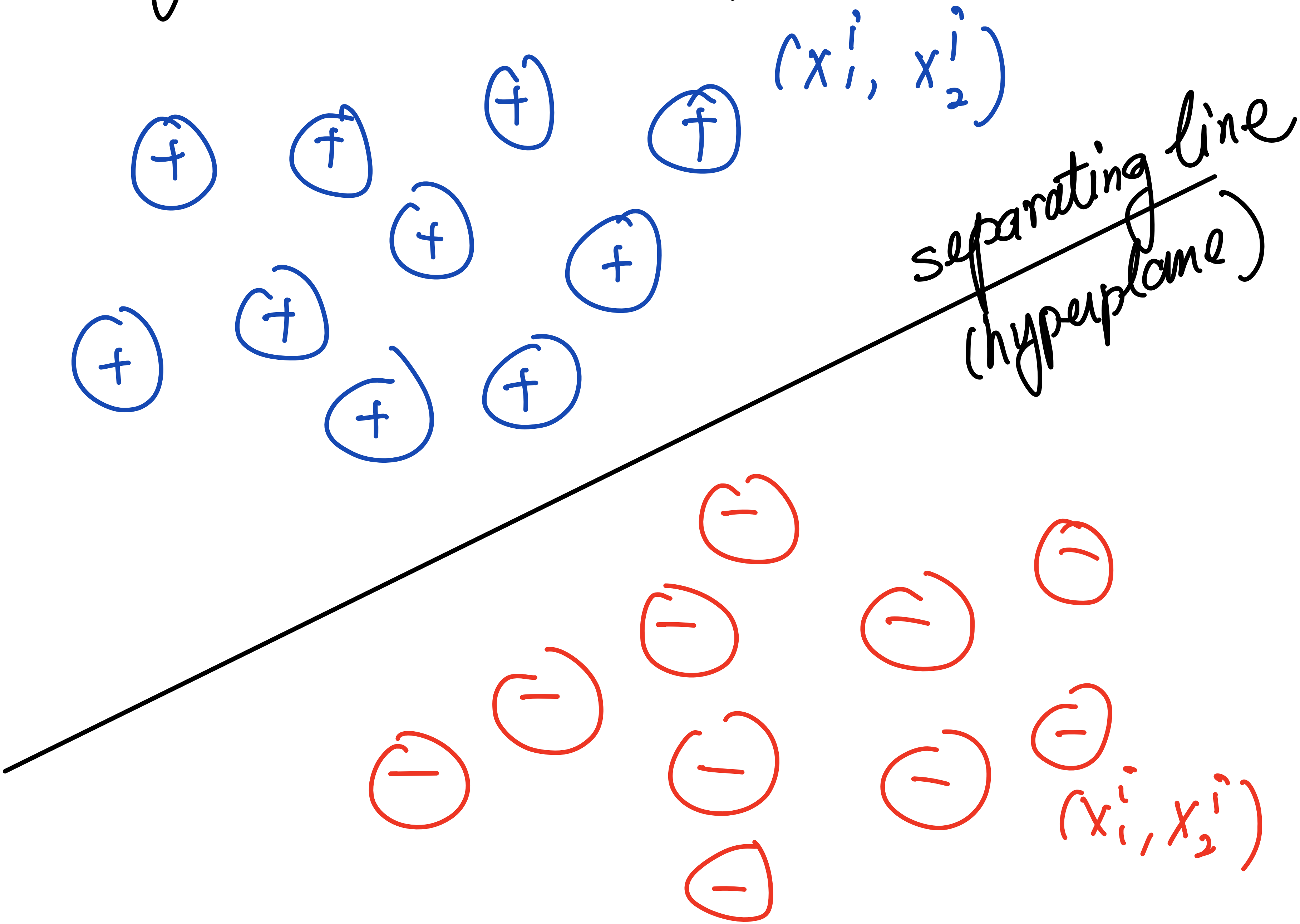
$$\boxed{\begin{aligned} &\min \quad \delta \\ &\text{s.t.} \quad -\delta \leq b_i - \sum_j a_{ij} x_j \leq \delta \end{aligned}}$$

$\left| b_i - \sum_j a_{ij} x_j \right| \leq \delta$ for all $i$

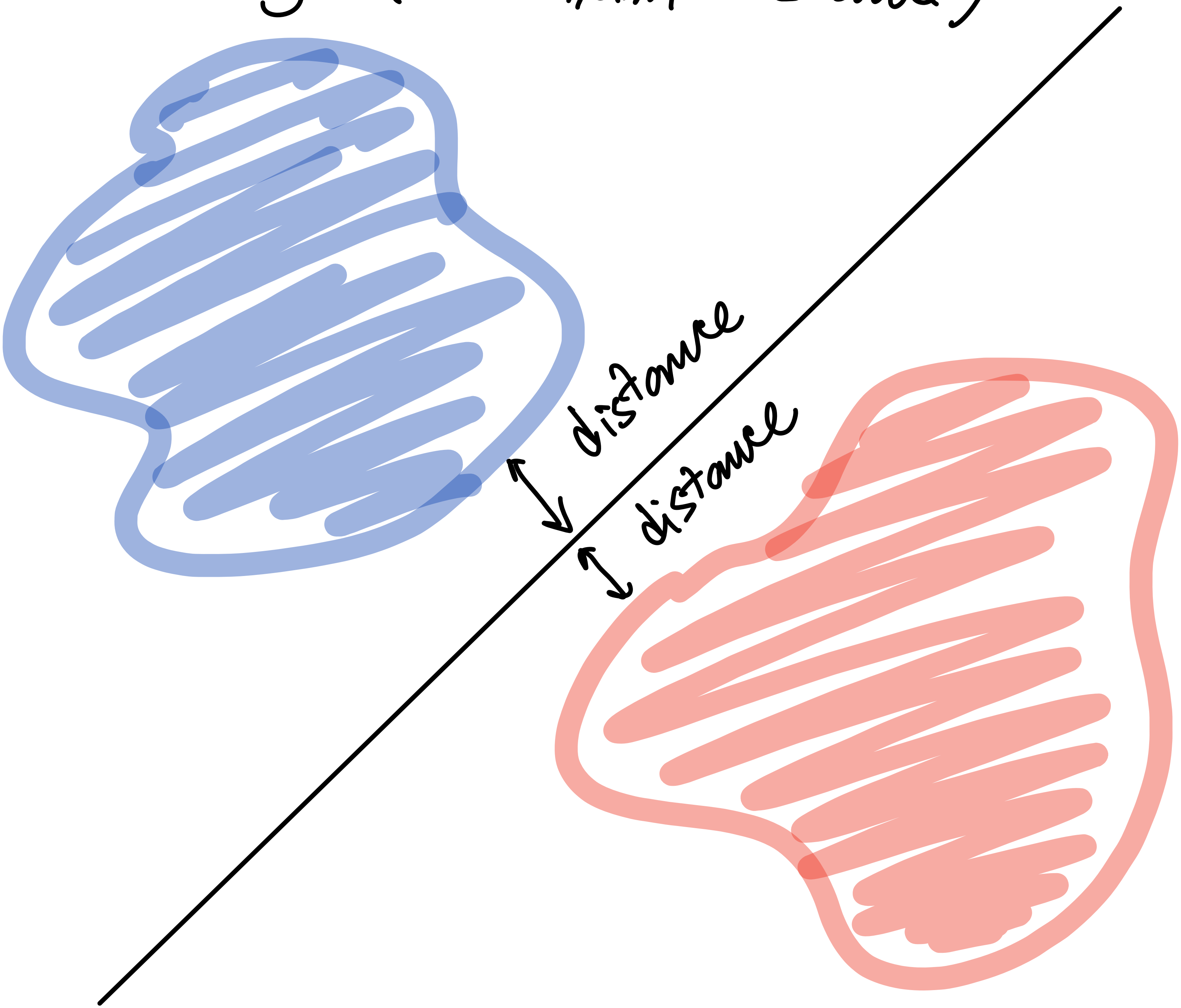
# Binary Classification given labelled points



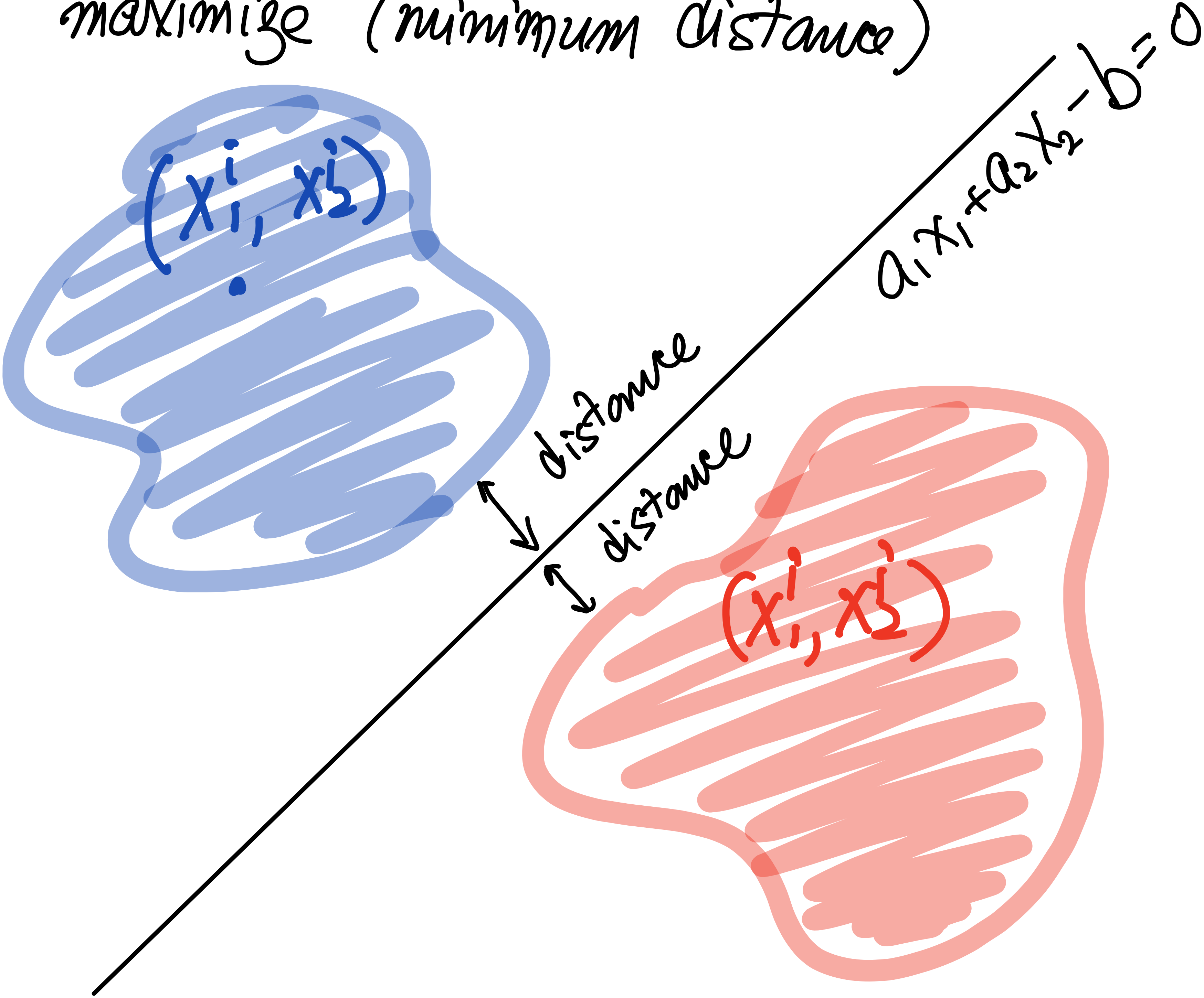
Binary Classification given labelled points



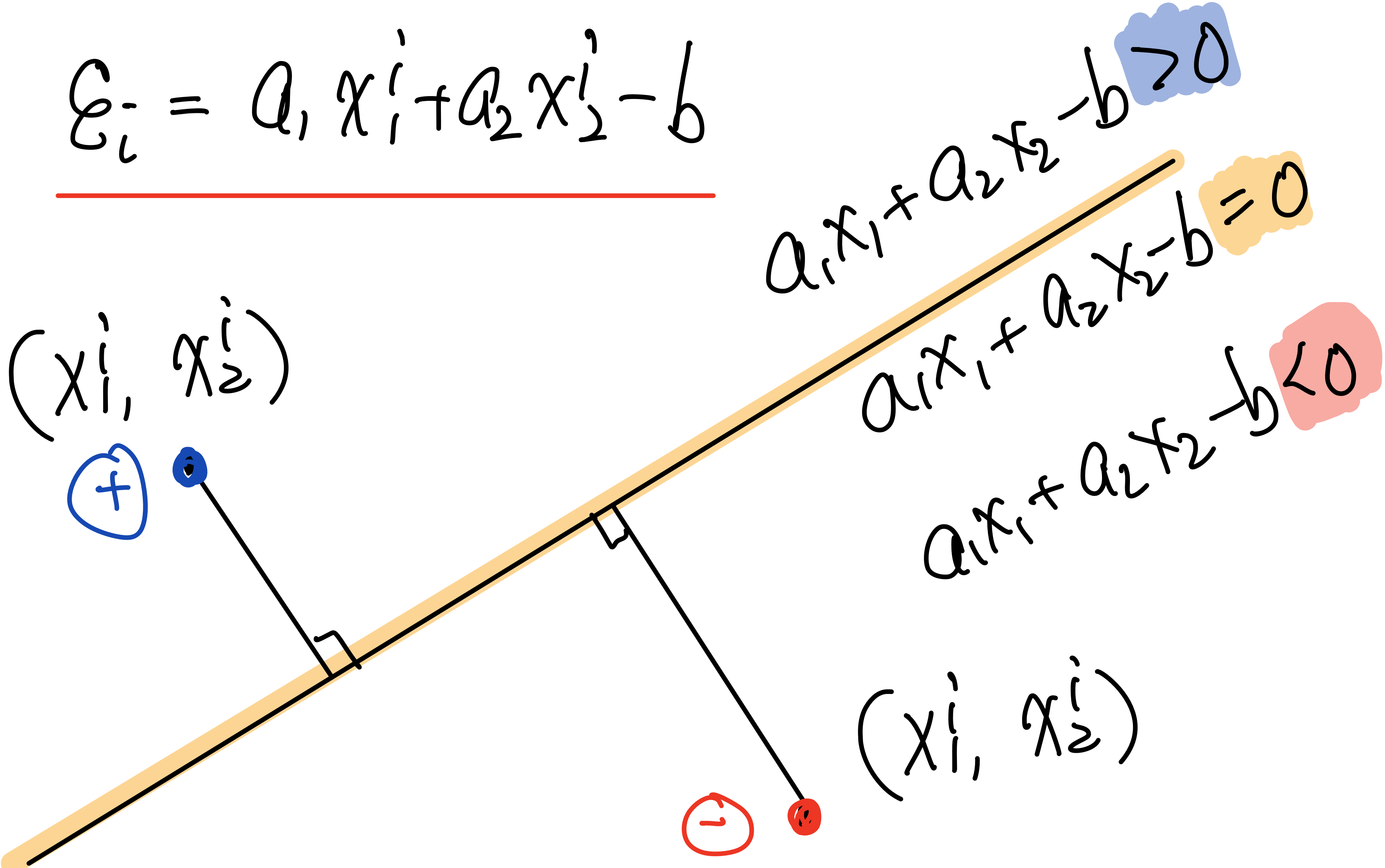
maximize (minimum distance)



maximize (minimum distance)



$$\varepsilon_i = a_1 x_1^i + a_2 x_2^i - b$$




$$\varepsilon_i = b - (a_1 x_1^i + a_2 x_2^i)$$

Distance of $\{\oplus\}$ from $a_1x_1+a_2x_2-b$

$$= \min_{i,\oplus}\left(a_1x_1^i+a_2x_2^i-b\right)$$

Distance of $\{\ominus\}$ from $a_1x_1+a_2x_2-b$

$$= \min_{i,\ominus}\left(b-\left(a_1x_1^i+a_2x_2^i\right)\right)$$

$$\max_{\delta,\, \varepsilon_i,\, a_1,\, a_2,\, b} \delta$$

s.t.

$$\varepsilon_i = a_1 x_1^i + a_2 x_2^i - b \quad \text{if } Label(i) = +$$

$$\varepsilon_i = b - a_1 x_1^i - a_2 x_2^i \quad \text{if } Label(i) = -$$

$$\delta \leq \varepsilon_i$$

$$\max_{\delta,\ \varepsilon_i,\ a_1,\ a_2,\ b} \delta$$

s.t.

$$\varepsilon_i = a_1 x_1^i + a_2 x_2^i - b \quad \text{if } Label(i) = \oplus$$

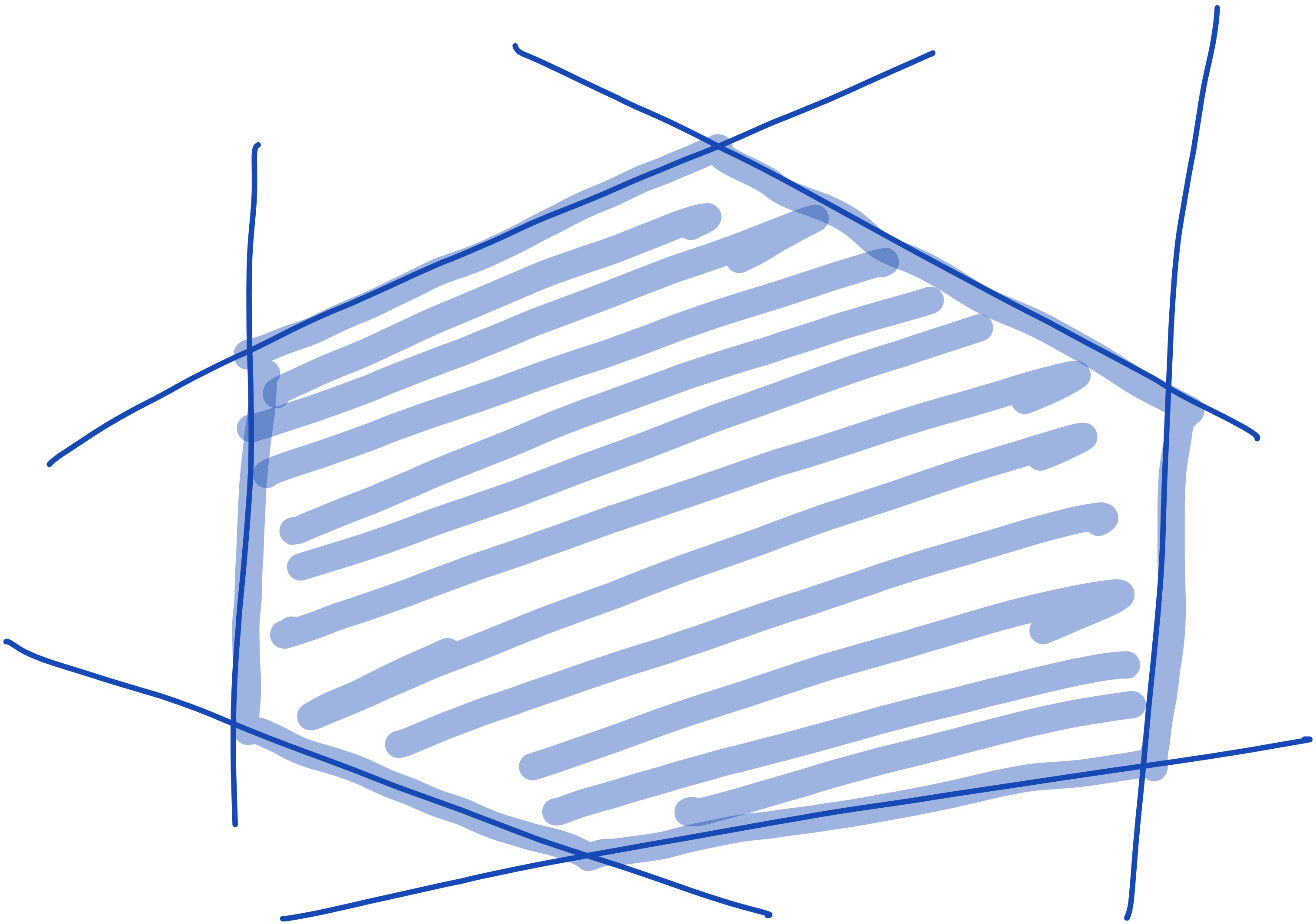
$$\varepsilon_i = b - a_1 x_1^i - a_2 x_2^i \quad \text{if } Label(i) = \ominus$$

$$\delta \leq \varepsilon_i$$

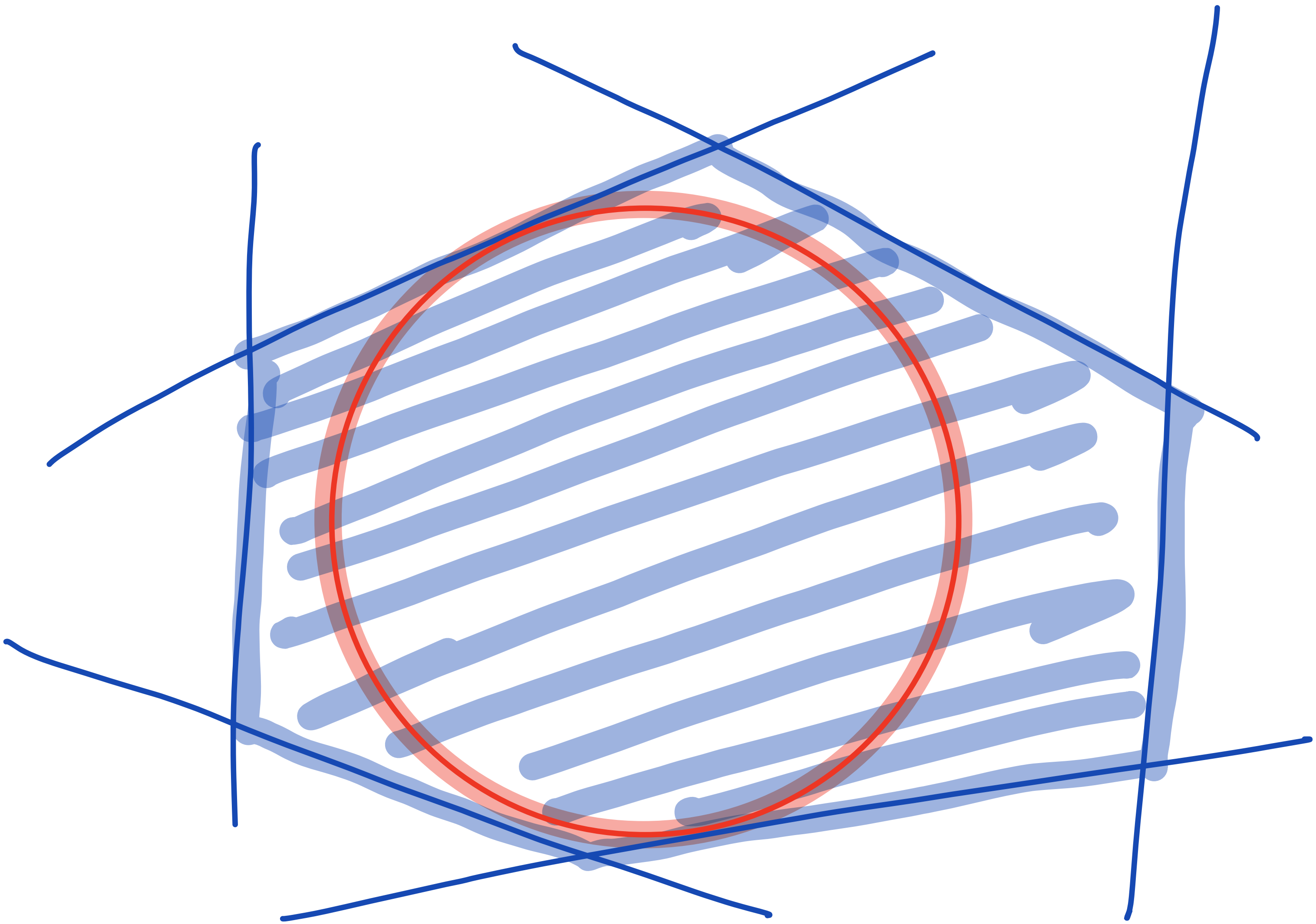
$$-1 \leq a_1 \leq 1, \quad -1 \leq a_2 \leq 1$$

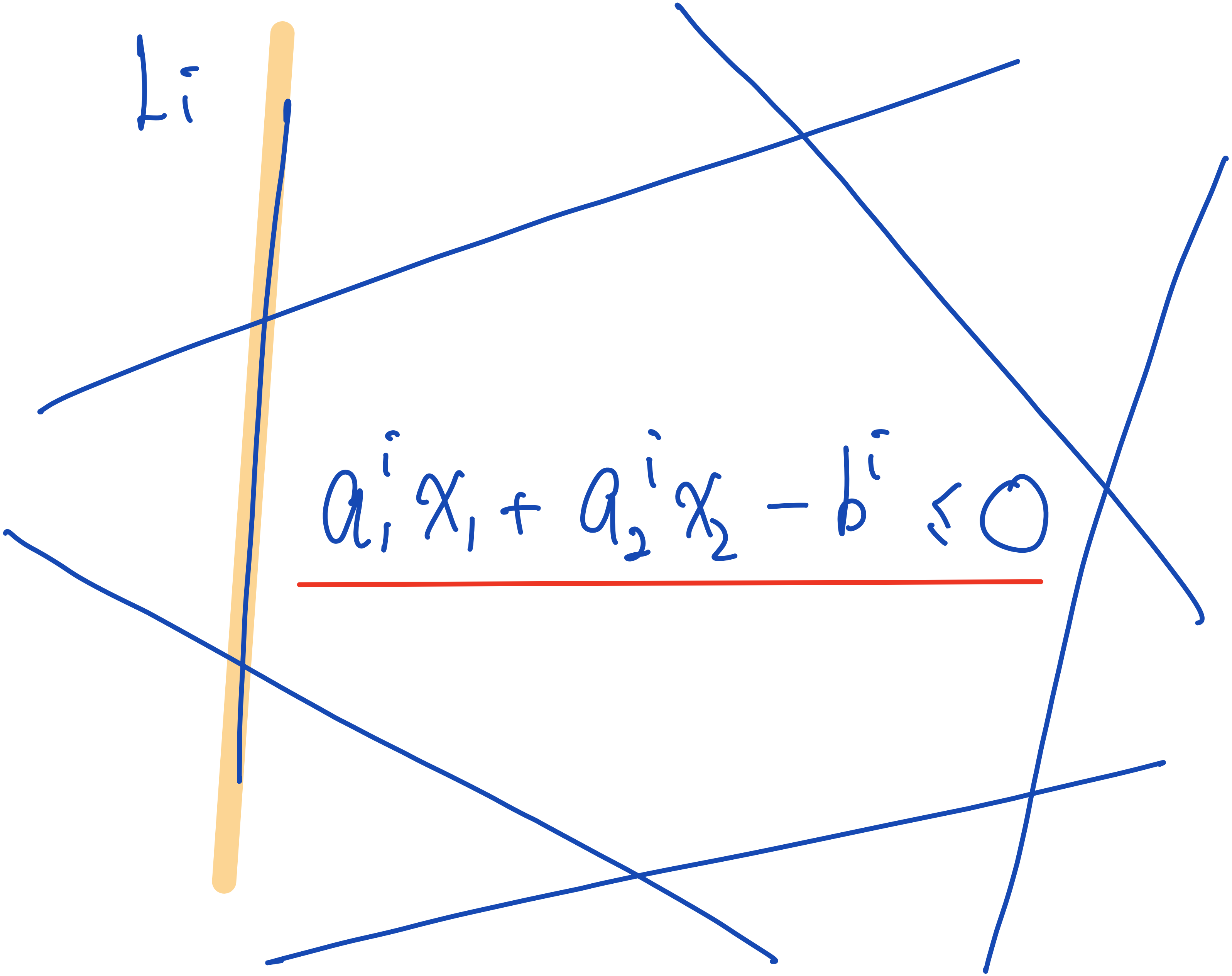
(normalization)

# Maximum Inscribed Circle in a Convex Polyhedron [MG, Sec 2.6]

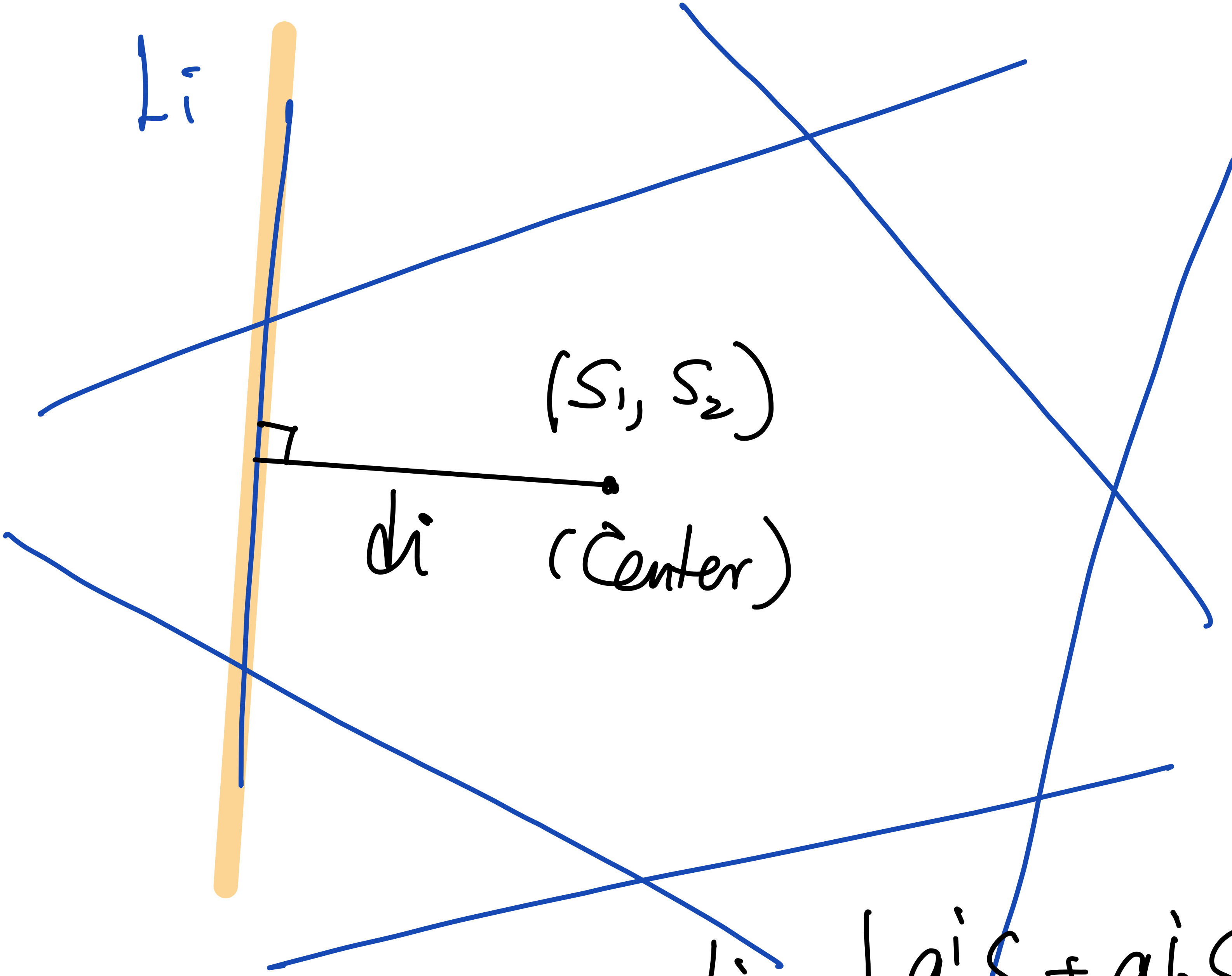


# Maximum Inscribed Circle in a Convex Polyhedron [MG, Sec 2.6]






$$a_1^i x_1 + a_2^i x_2 - b^i \le 0$$




$$d_i = \left| \frac{a^i_1 s_1 + a^i_2 s_2 - b^i}{\sqrt{(a^i_1)^2 + (a^i_2)^2}} \right|$$

$$\underset{(s_1, s_2, d)}{\text{maximize}} \quad d$$

$$\text{s.t.} \quad d \le d_i = \left| \frac{a_1^i s_1 + a_2^i s_2 - b^i}{\sqrt{(a_1^i)^2 + (a_2^i)^2}} \right|$$

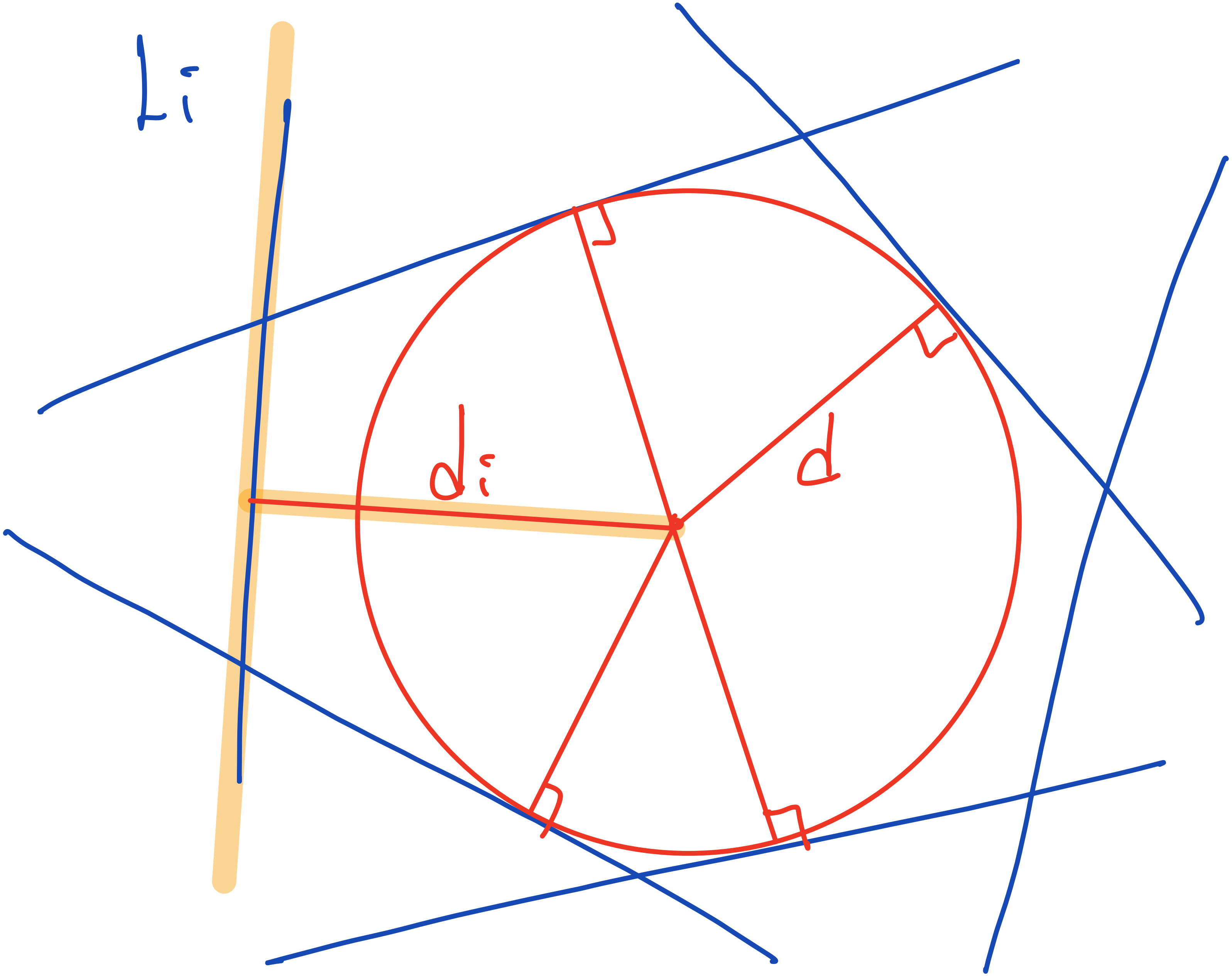
$$a_1^i s_1 + a_2^i s_2 - b_i \le 0$$

$$\underset{(s_1, s_2, d)}{\text{maximize}} \quad d$$

$$\text{s.t.} \quad d \leq \frac{b^i - a_1^i s_1 - a_2^i s_2}{\sqrt{(a_1^i)^2 + (a_2^i)^2}}$$

$$\cancel{a_1^i s_1 + a_2^i s_2 - b_i \leq 0}$$

redundant



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[MG]

# 8.7 Smallest Balls and Convex Programming

**The smallest ball problem.** We are given points $\mathbf{p}_1, \ldots, \mathbf{p}_n \in \mathbb{R}^d$, and we want to find a ball of the smallest radius that contains all the points.[8]