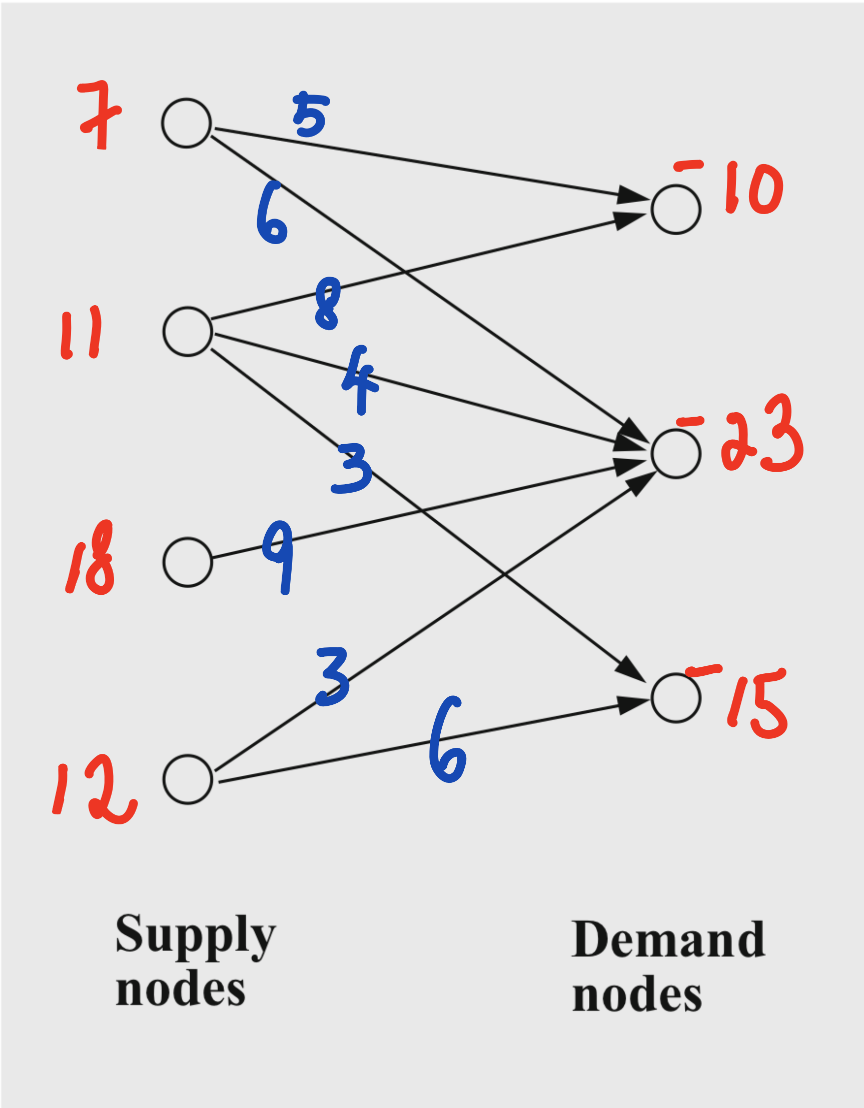
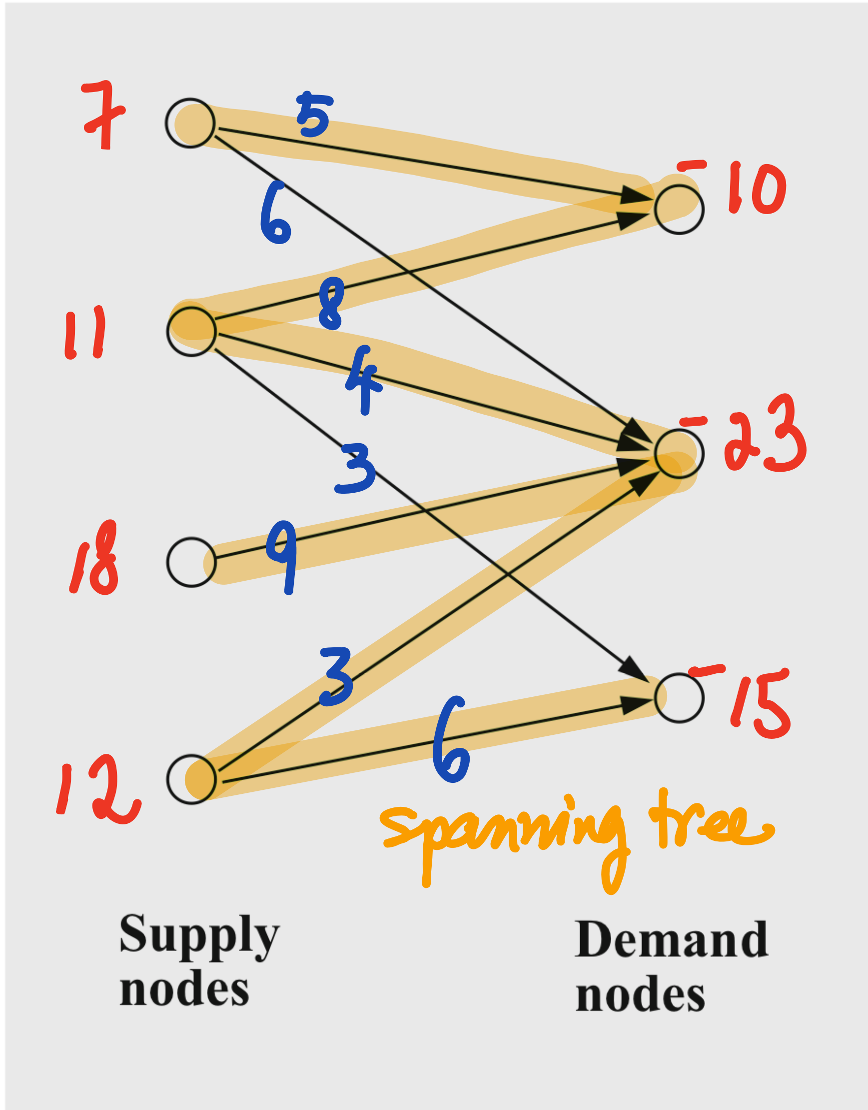
Transportation Problem [V] p. 258



Transportation Problem [V] p.258



c_{ij}	-10	-23	-15
7	5	6	*
11	8	4	3
18	*	9	*
12	*	3	6

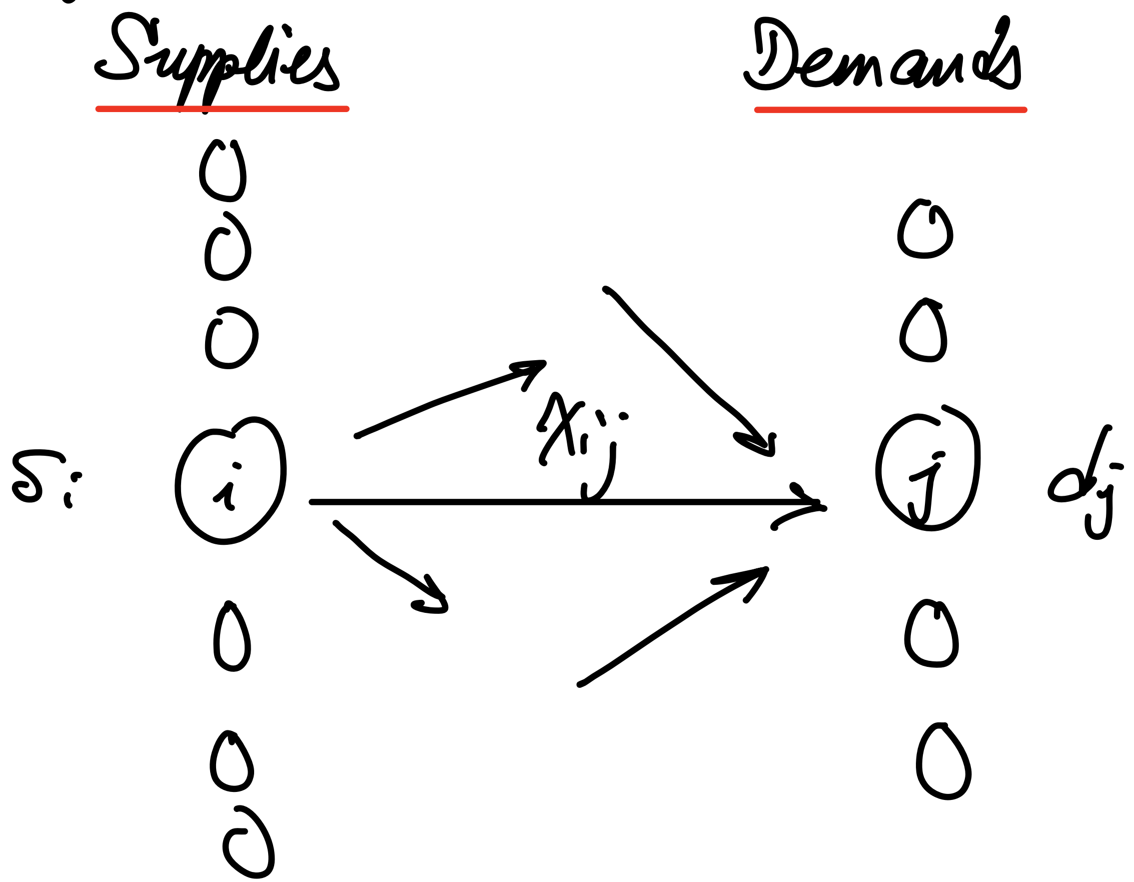
z_{ij}

$y_i \backslash y_j$	5	1	4
0	$\boxed{7}$	5	*
-3	$\boxed{3}$	$\boxed{8}$	-4
-8	*	$\boxed{18}$	*
-2	*	$\boxed{-3}$	$\boxed{15}$

x_{ij}

Variants of Network Flows

Inequality constraints (Transportation Problem) [C] p. 320

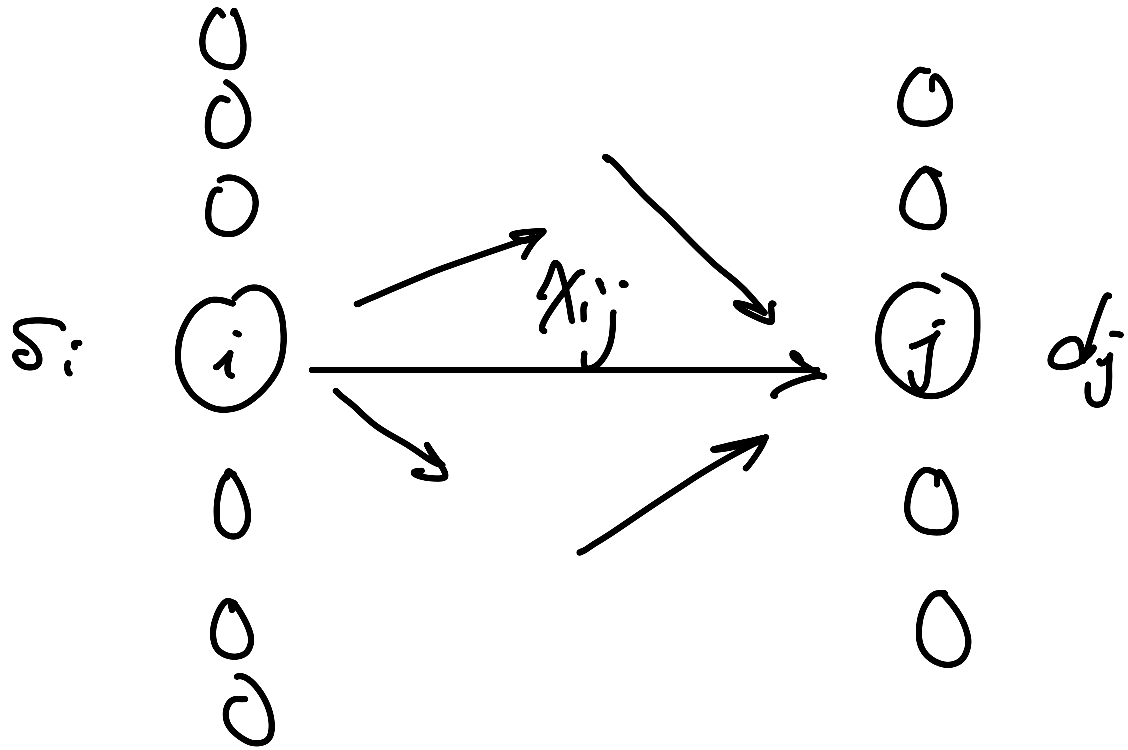


$$\sum_j x_{ij} = s_i \,, \qquad \sum_i x_{ij} = d_j$$

Variants of Network Flows

Inequality constraints (Transportation Problem)

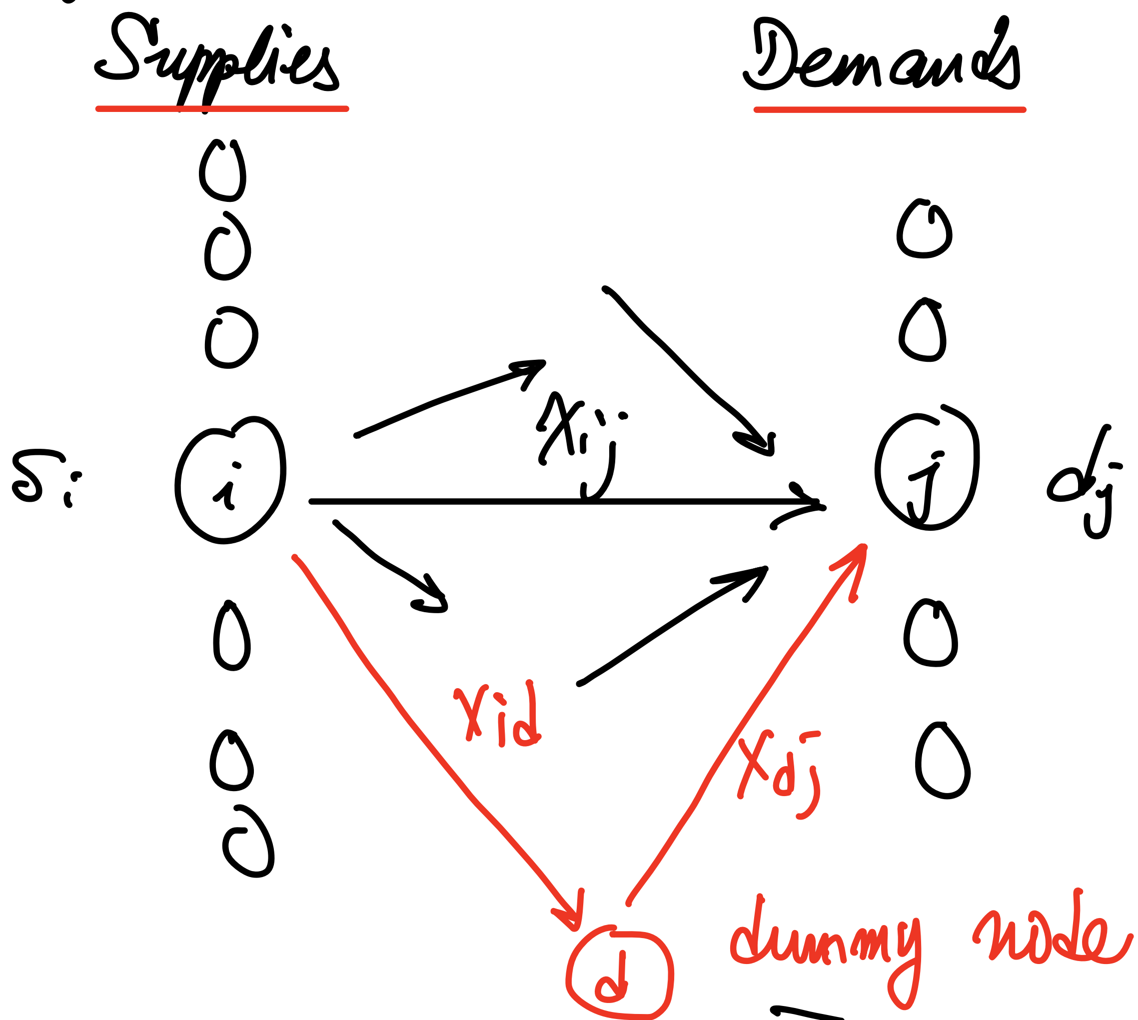
Supplies **Demands**



$$\sum_j x_{ij} \leq s_i, \qquad \sum_i x_{ij} \geq d_j$$

Variants of Network Flows

Inequality constraints (Transportation Problem)



$x_{id}, x_{dj} \geq 0$

$$\sum_j x_{ij} + \underline{x_{id}} = s_i, \qquad \sum_i x_{ij} = d_j + \underline{x_{dj}}$$

Variants of Network Flows

Inequality constraints (Transportation Problem)

$$\sum_j x_{ij} + \underline{x_{id}} = s_i, \qquad \sum_i x_{ij} = d_j + \underline{x_{dj}}$$

$$\underline{\sum_{i,j} x_{ij}} + \sum_i x_{id} = \sum_i s_i \qquad \underline{\sum_{j,i} x_{ij}} = \sum_j d_j + \sum_j x_{dj}$$

$=$

Hence, the new additional constraint:

$$\boxed{\sum_i s_i - \sum_i x_{id} = \sum_j d_j + \sum_j x_{dj}}$$

Inequality constraints (Transportation Problem) [c] p.321

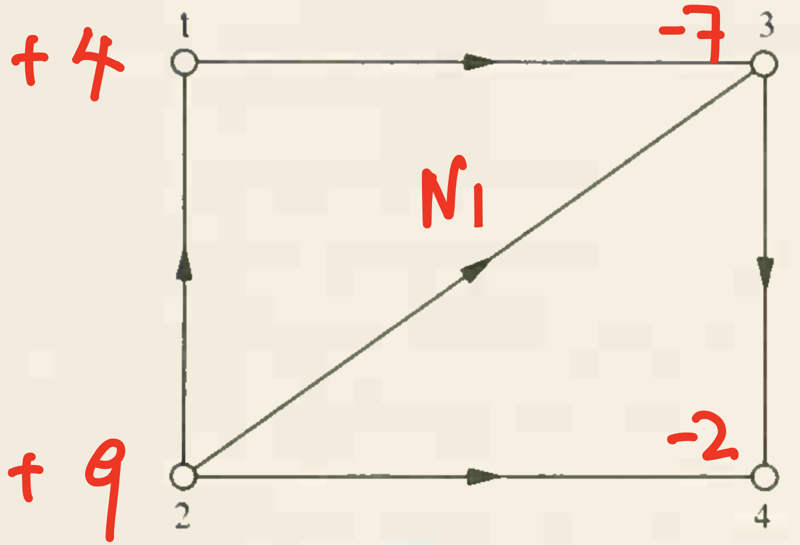


Figure 20.1

4 units at the source 1

9 units at the source 2

and demands of

7 units at the sink 3

2 units at the sink 4.

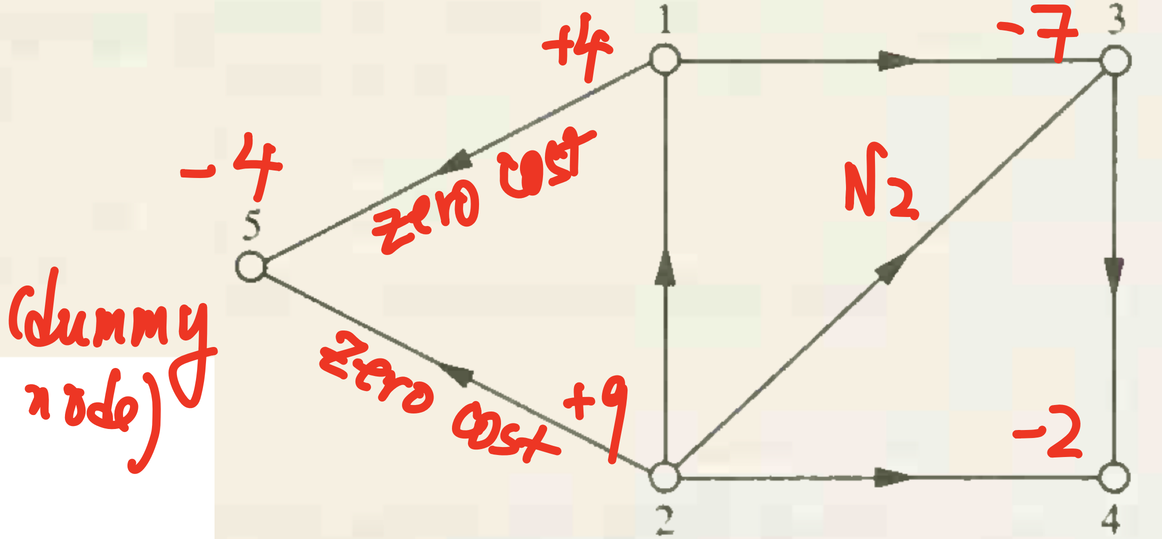


Figure 20.2

Inequality constraints (Transportation Problem) [c] p. 321

$$
\begin{array}{ll}
\text{minimize} & z = c_{13}x_{13} + c_{21}x_{21} + c_{23}x_{23} + c_{24}x_{24} + c_{34}x_{34} \\
\text{subject to} & -x_{13} + x_{21} \geq -4 \\
 & -x_{21} - x_{23} - x_{24} \geq -9 \\
 & x_{13} + x_{23} - x_{34} = 7 \\
 & x_{24} + x_{34} = 2 \\
 & x_{13}, x_{21}, x_{23}, x_{24}, x_{34} \geq 0
\end{array}
$$

N_1

$$
\begin{array}{ll}
\text{minimize} & z \\
\text{subject to} & -x_{13} + x_{21} - x_{15} = -4 \\
 & -x_{21} - x_{23} - x_{24} - x_{25} = -9 \\
 & x_{13} + x_{23} - x_{34} = 7 \\
 & x_{24} + x_{34} = 2 \\
 & x_{15} + x_{25} = 4 \\
 & x_{13}, x_{21}, x_{23}, x_{24}, x_{34}, x_{15}, x_{25} \geq 0.
\end{array}
$$

N_2

Inequality constraints (More generally, [C] p.322)

$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ - incident matrix

$$A_1 \vec{X} \geq -\vec{b}_1$$

$$A_2 \vec{X} \leq -\vec{b}_2$$

$$A_1 \vec{X} = -\vec{b}_1 + \vec{W}_1, \quad \vec{W}_1 \geq 0$$

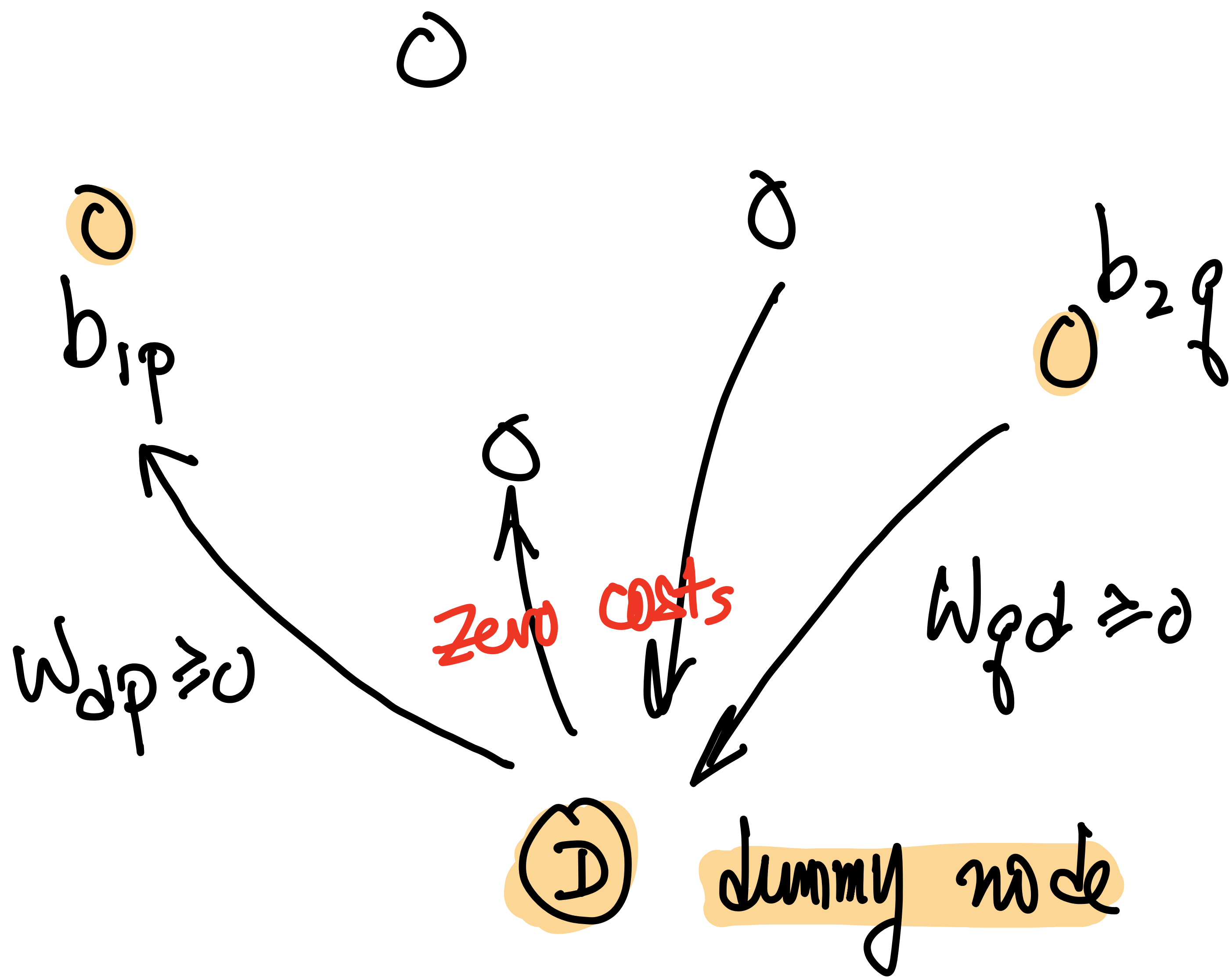
$$A_2 \vec{X} = -\vec{b}_2 - \vec{W}_2, \quad \vec{W}_2 \geq 0$$

$+$

new constraint

$$\boxed{\sum_p W_{1p} - \sum_q W_{2q} = \sum_p b_{1p} + \sum_q b_{1q}}$$

Inequality constraints (More generally, [c] p.322)



Upper Bounded Transshipment Problem [V]

p. 263

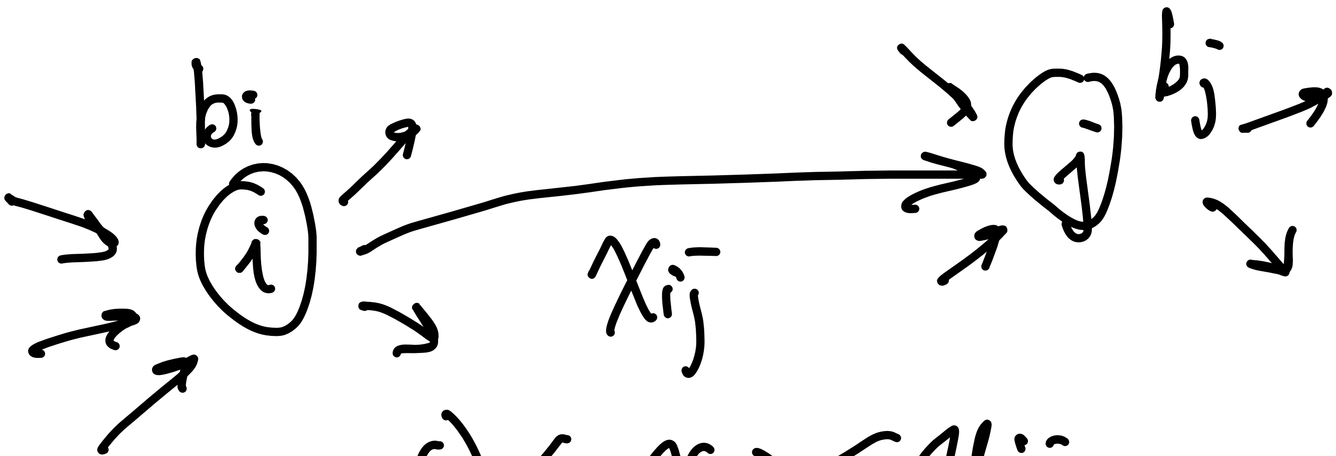
$$AX = b, \qquad 0 \leq X \leq U$$

$$(i) \xrightarrow{x_{ij}} (j)$$

$$0 \leq x_{ij} \leq u_{ij}$$

Upper Bounded Transhipment Problem

$$AX = -b, \qquad 0 \le X \le u$$



$$0 \le x_{ij} \le u_{ij}$$

1 at i $\qquad \cdots\cdots - x_{ij} \cdots = -b_i$

2 at j $\qquad \cdots\cdots + x_{ij} \cdots = -b_j$

3 $\qquad x_{ij} + t_{ij} = u_{ij} \quad (t_{ij} \ge 0)$

Upper Bounded Transhipment Problem

(1) $\cdots\cdots - x_{ij} \cdots\cdots = -b_i$ at (i)

(2) $\cdots\cdots + x_{ij} \cdots\cdots = -b_j$ at (j)

(3) $x_{ij} + t_{ij} = u_{ij}$

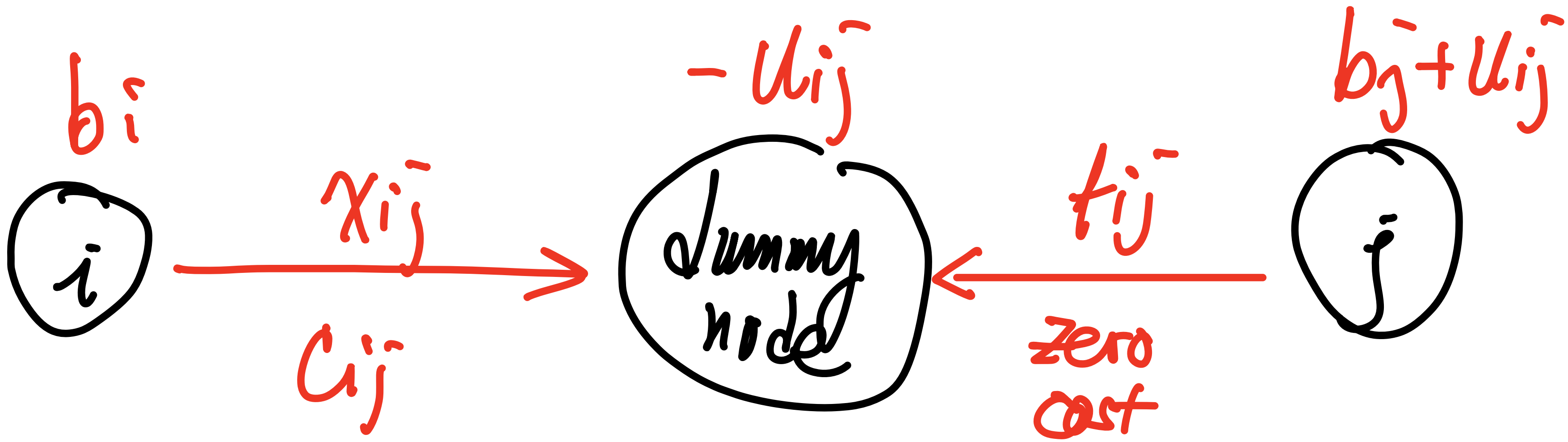
(1) $\cdots\cdots - x_{ij} \cdots\cdots = -b_i$

(2)–(3) $\cdots\cdots\cdots - t_{ij} = -b_j - u_{ij}$

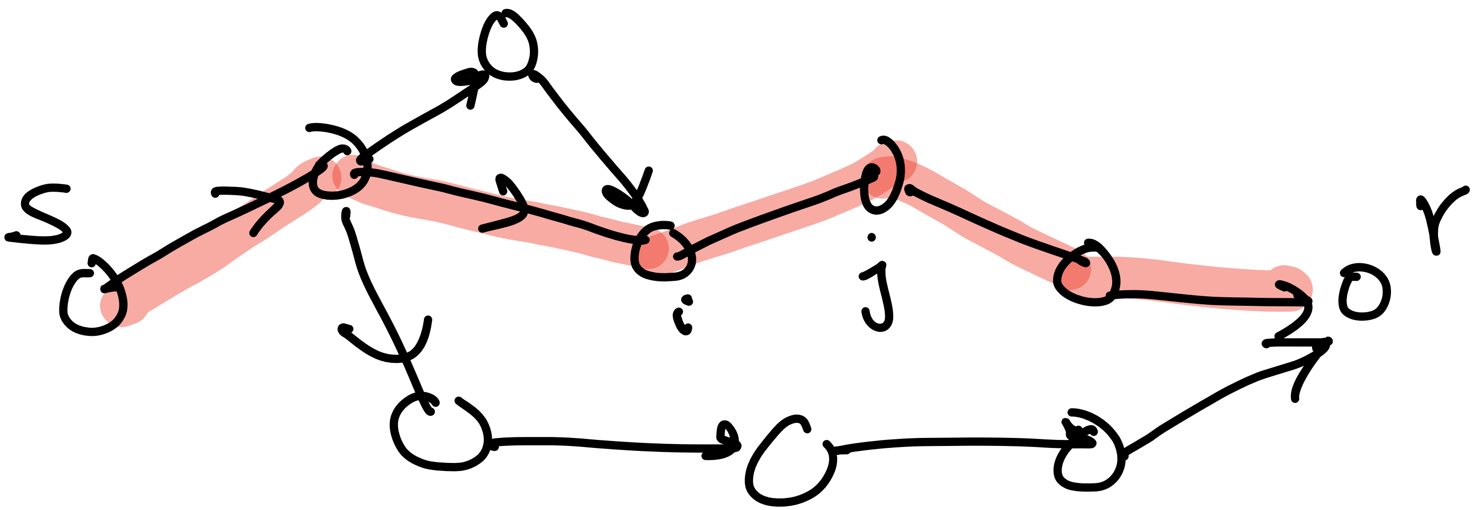
(3) $x_{ij} + t_{ij} = u_{ij}$

Upper Bounded Transhipment Problem

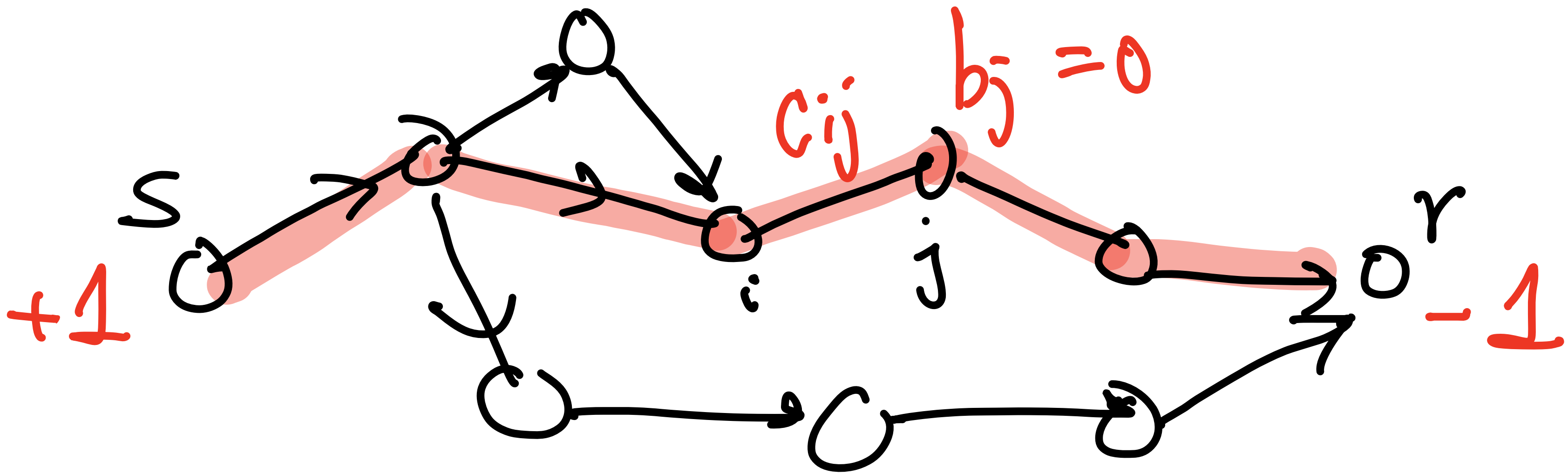
$$
\begin{cases}
(1) \quad \cdots \; -x_{ij} \; \cdots = -b_i \\
(2)' \quad \cdots \; -t_{ij} = -b_j - u_{ij} \\
(3) \quad x_{ij} + t_{ij} = u_{ij}
\end{cases}
$$



Shortest Path Problem



Shortest Path Problem



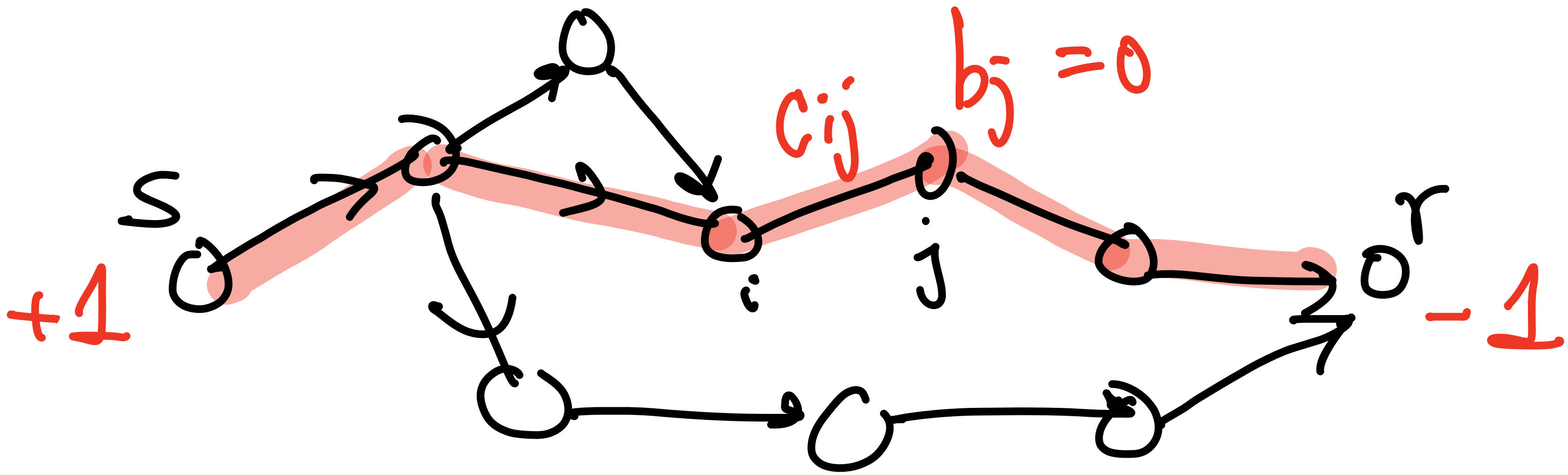
Network flow formulation

$$\min \sum_{i,j} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j} x_{sj} = 1, \quad \sum_{i} x_{ir} = 1$$

$(x_{ij} \leq \geq 0)$ $\quad \sum_{i} x_{ik} - \sum_{j} x_{kj} = 0, \quad k \neq s, r$

Shortest Path Problem



Network flow formulation

($x_{ij} \in \{0, 1\}$?)
(Uniqueness ?)

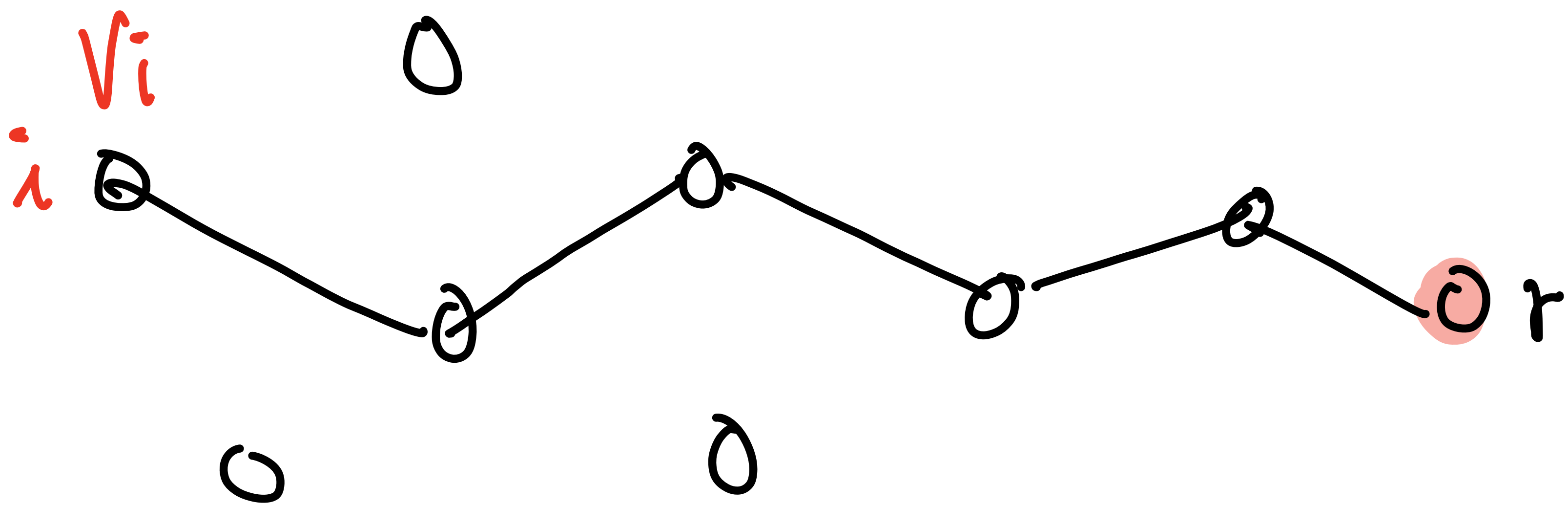
$$\min \sum_{i,j} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_j x_{sj} = 1, \quad \sum_i x_{ir} = 1$$

$$\sum_i x_{ik} - \sum_j x_{kj} = 0, \quad k \neq s, r$$

($x_{ij} \geq 0$)

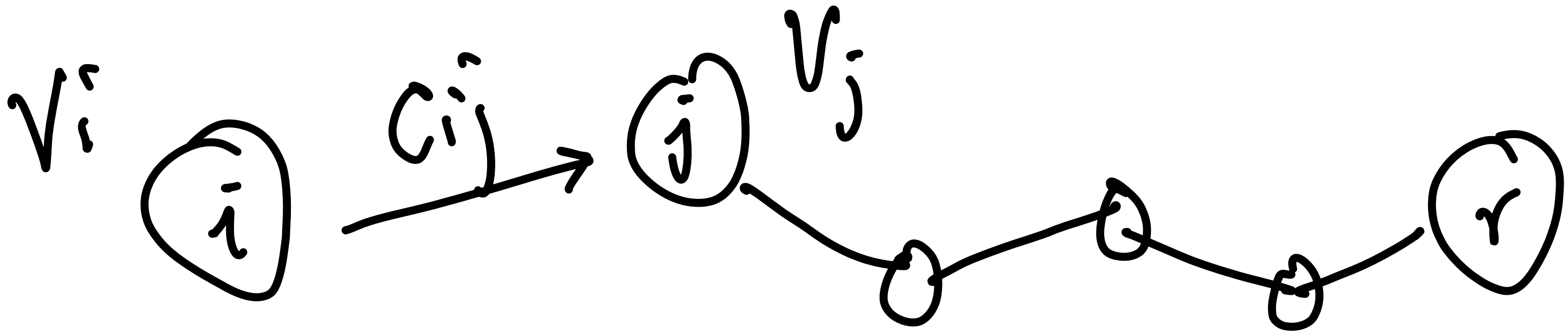
Shortest Path Problem (Bellman's Eqn)



$$\boxed{V_i = \text{shortest distance from } i \text{ to } r}$$

label

Shortest Path Problem (Bellman's Eqn)



$$\boxed{\begin{aligned} V_i &= \min_j \{c_{ij} + V_j\} \\ V_r &= 0 \end{aligned}}$$

Shortest Path Problem (Bellman's Eqn)

$$\boxed{V_i = \min_j \{c_{ij} + V_j\}, \quad V_r = 0}$$

Approximation / Iteration

$k=0 \qquad V_r^{(0)} = 0, \qquad V_i^{(0)} = +\infty, \quad i \neq r$

$k \geq 0 \qquad V_r^{(k)} = 0, \qquad V_i^{(k+1)} = \min_j \{c_{ij} + V_j^{(k)}\},$

Shortest Path Problem (Bellman's Eqn)

$$\boxed{V_i = \min_j \{ c_{ij} + V_j \}, \quad V_r = 0}$$

Approximation / Iteration

$k = 0 \qquad V_r^{(0)} = 0, \qquad V_i^{(0)} = +\infty, \quad i \neq r$

$k \geq 0 \qquad V_r^{(k)} = 0, \quad V_i^{(k+1)} = \min_j \{ c_{ij} + V_j^{(k)} \},$

- $V_i^{(k)}$ = shortest path from i to r, with $\leq k$ arcs.
- Needs at most m iterations, m = # of nodes

Shortest Path Problem (Dijkstra Alg.)

J – set of finished node

v_j – label of j (shortest distance j to r)

h_j – node to visit next in shortest path

Shortest Path Problem (Dijkstra Alg.)

Initialize:

$$\mathcal{F} = \emptyset$$

$$v_j = \begin{cases} 0 & j = r, \\ \infty & j \neq r. \end{cases}$$

while $(|\mathcal{F}^c| > 0)\{$

$\quad j = \text{argmin}\{v_k : k \notin \mathcal{F}\}$

$\quad \mathcal{F} \leftarrow \mathcal{F} \cup \{j\}$

$\quad$ for each i for which $(i, j) \in \mathcal{A}$ and $i \notin \mathcal{F}$ $\{$

$\quad\quad$ if $(c_{ij} + v_j < v_i)$ $\{$

$\quad\quad\quad v_i = c_{ij} + v_j$

$\quad\quad\quad h_i = j$

$\quad\quad \}$

$\quad \}$

$\}$