2.1 Optimized Diet: Wholesome and Cheap?

> ... and when Rabbit said, "Honey or condensed milk
> with your bread?" he was so excited that he said,
> "Both," and then, so as not to seem greedy, he added,
> "But don't bother about the bread, please."
>
> *A. A. Milne, Winnie the Pooh*

The Office of Nutrition Inspection of the EU recently found out that dishes
served at the dining and beverage facility "Bullneck's," such as herring, hot
dogs, and house-style hamburgers do not comport with the new nutritional
regulations, and its report mentioned explicitly the lack of vitamins A and
C and dietary fiber. The owner and operator of the aforementioned facility
is attempting to rectify these shortcomings by augmenting the menu with
vegetable side dishes, which he intends to create from white cabbage, carrots,
and a stockpile of pickled cucumbers discovered in the cellar. The following
table summarizes the numerical data: the prescribed amount of the vitamins
and fiber per dish, their content in the foods, and the unit prices of the foods.[1]

Food	Carrot, Raw	White Cabbage, Raw	Cucumber, Pickled	Required per dish
Vitamin A [mg/kg]	35	0.5	0.5	0.5 mg
Vitamin C [mg/kg]	60	300	10	15 mg
Dietary Fiber [g/kg]	30	20	10	4 g
price [€/kg]	0.75	0.5	0.15*	—

*Residual accounting price of the inventory, most likely unsaleable.

At what minimum additional price per dish can the requirements of the
Office of Nutrition Inspection be satisfied? This question can be expressed
by the following linear program:

$$\begin{aligned}
\text{Minimize} \quad & 0.75x_1 + 0.5x_2 + 0.15x_3 \\
\text{subject to} \quad & x_1 \geq 0 \\
& x_2 \geq 0 \\
& x_3 \geq 0 \\
& 35x_1 + 0.5x_2 + 0.5x_3 \geq 0.5 \\
& 60x_1 + 300x_2 + 10x_3 \geq 15 \\
& 30x_1 + 20x_2 + 10x_3 \geq 4.
\end{aligned}$$

The variable x_1 specifies the amount of carrot (in kg) to be added to each dish,
and similarly for x_2 (cabbage) and x_3 (cucumber). The objective function

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[1] For those interested in healthy diet: The vitamin contents and other data are
more or less realistic.

expresses the price of the combination. The amounts of carrot, cabbage, and cucumber are always nonnegative, which is captured by the conditions $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ (if we didn't include them, an optimal solution might perhaps have the amount of carrot, say, negative, by which one would seemingly save money). Finally, the inequalities in the last three lines force the requirements on vitamins A and C and of dietary fiber.

The linear program can be solved by standard methods. The optimal solution yields the price of € 0.07 with the following doses: carrot 9.5 g, cabbage 38 g, and pickled cucumber 290 g per dish (all rounded to two significant digits). This probably wouldn't pass another round of inspection. In reality one would have to add further constraints, for example, one on the maximum amount of pickled cucumber.

We have included this example so that our treatment doesn't look too revolutionary. It seems that all introductions to linear programming begin with various dietary problems, most likely because the first large-scale problem on which the simplex method was tested in 1947 was the determination of an adequate diet of least cost. Which foods should be combined and in what amounts so that the required amounts of all essential nutrients are satisfied and the daily ration is the cheapest possible. The linear program had 77 variables and 9 constraints, and its solution by the simplex method using hand-operated desk calculators took approximately 120 man-days.

Later on, when George Dantzig had already gained access to an electronic computer, he tried to optimize his own diet as well. The optimal solution of the first linear program that he constructed recommended daily consumption of several liters of vinegar. When he removed vinegar from the next input, he obtained approximately 200 bouillon cubes as the basis of the daily diet. This story, whose truth is not entirely out of the question, doesn't diminish the power of linear programming in any way, but it illustrates how difficult it is to capture mathematically all the important aspects of real-life problems. In the realm of nutrition, for example, it is not clear even today what exactly the influence of various components of food is on the human body. (Although, of course, many things *are* clear, and hopes that the science of the future will recommend hamburgers as the main ingredient of a healthy diet will almost surely be disappointed.) Even if it were known perfectly, few people want and can formulate exactly what they expect from their diet—apparently, it is much easier to formulate such requirements for the diet of someone else. Moreover, there are nonlinear dependencies among the effects of various nutrients, and so the dietary problem can never be captured perfectly by linear programming.

There are many applications of linear programming in industry, agriculture, services, etc. that from an abstract point of view are variations of the diet problem and do not introduce substantially new mathematical tricks. It may still be challenging to design good models for real-life problems of this kind, but the challenges are not mathematical. We will not dwell on