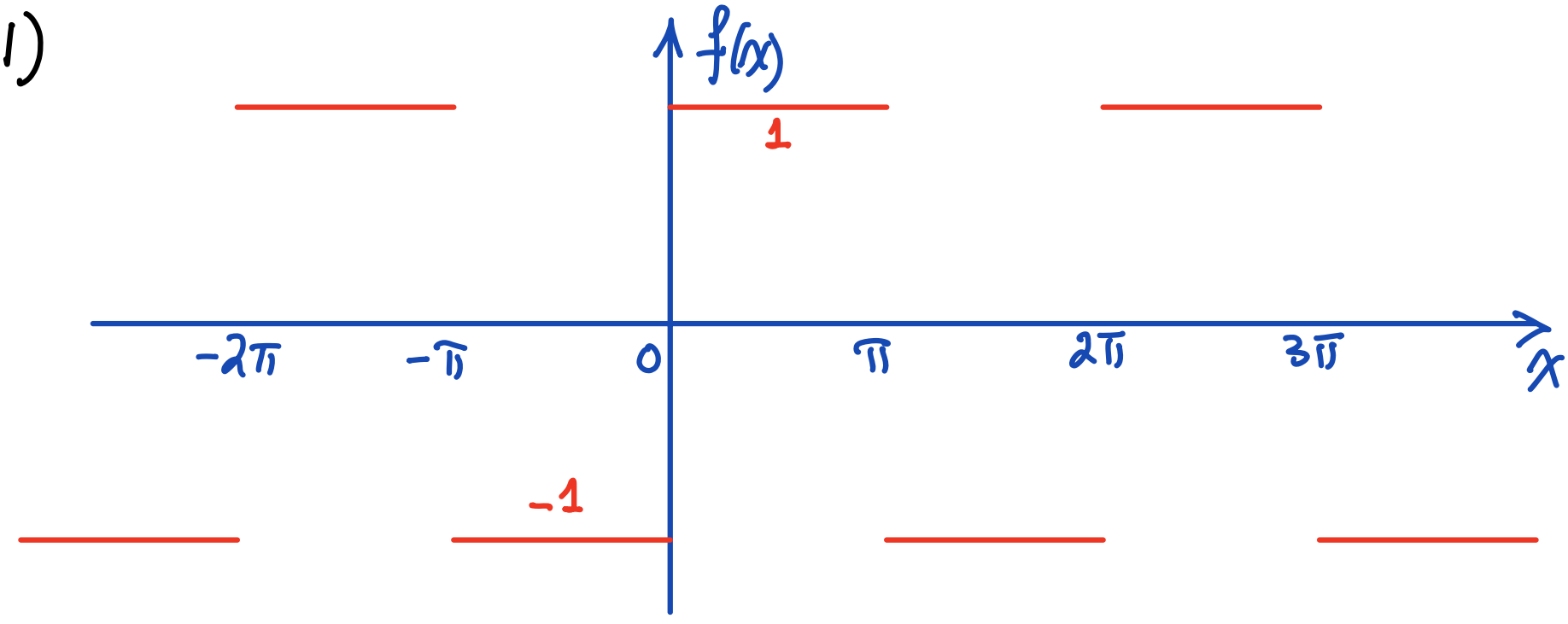
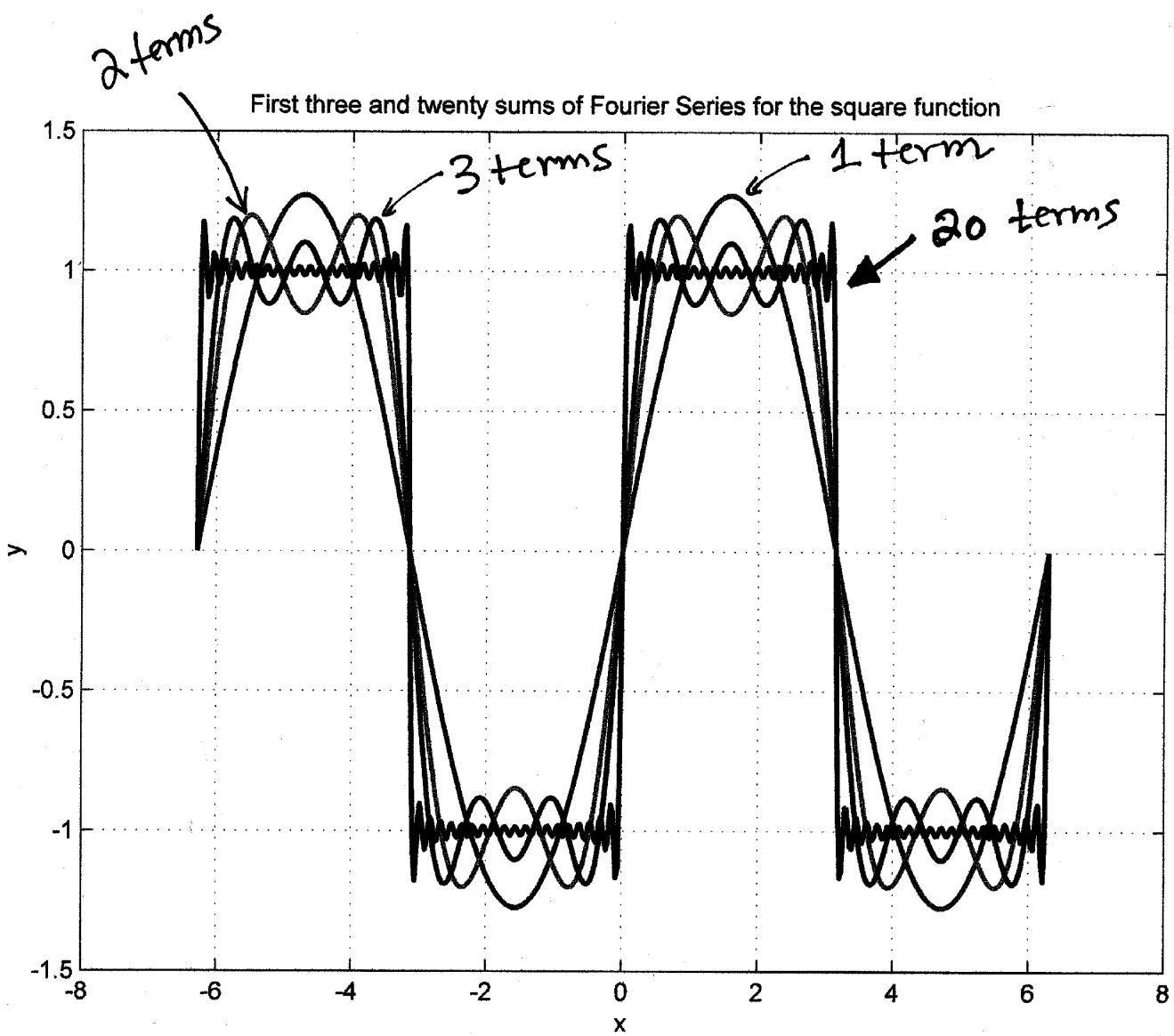
Examples of Fourier Series

(1)



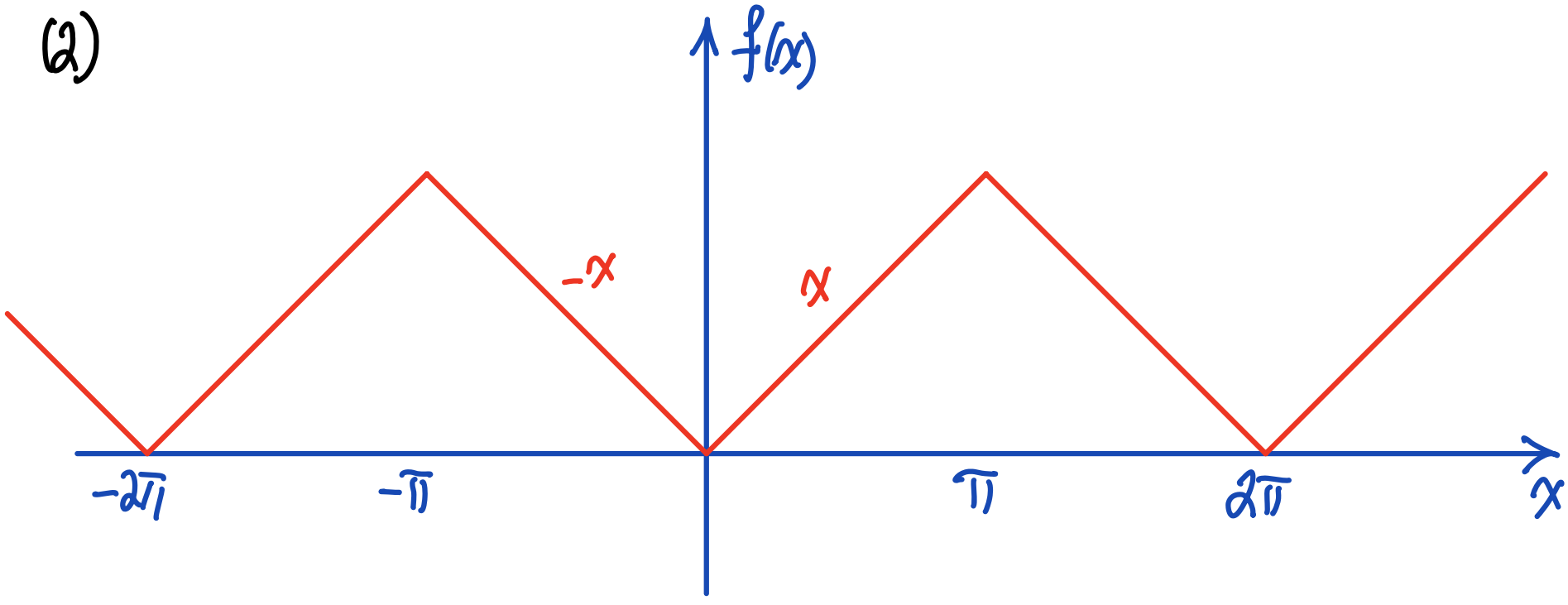
$$f(x)=\begin{cases}1 & 0<x<\pi\\ -1 & -\pi<x<0\end{cases} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}$$

p.26 (6)



Examples of Fourier Series

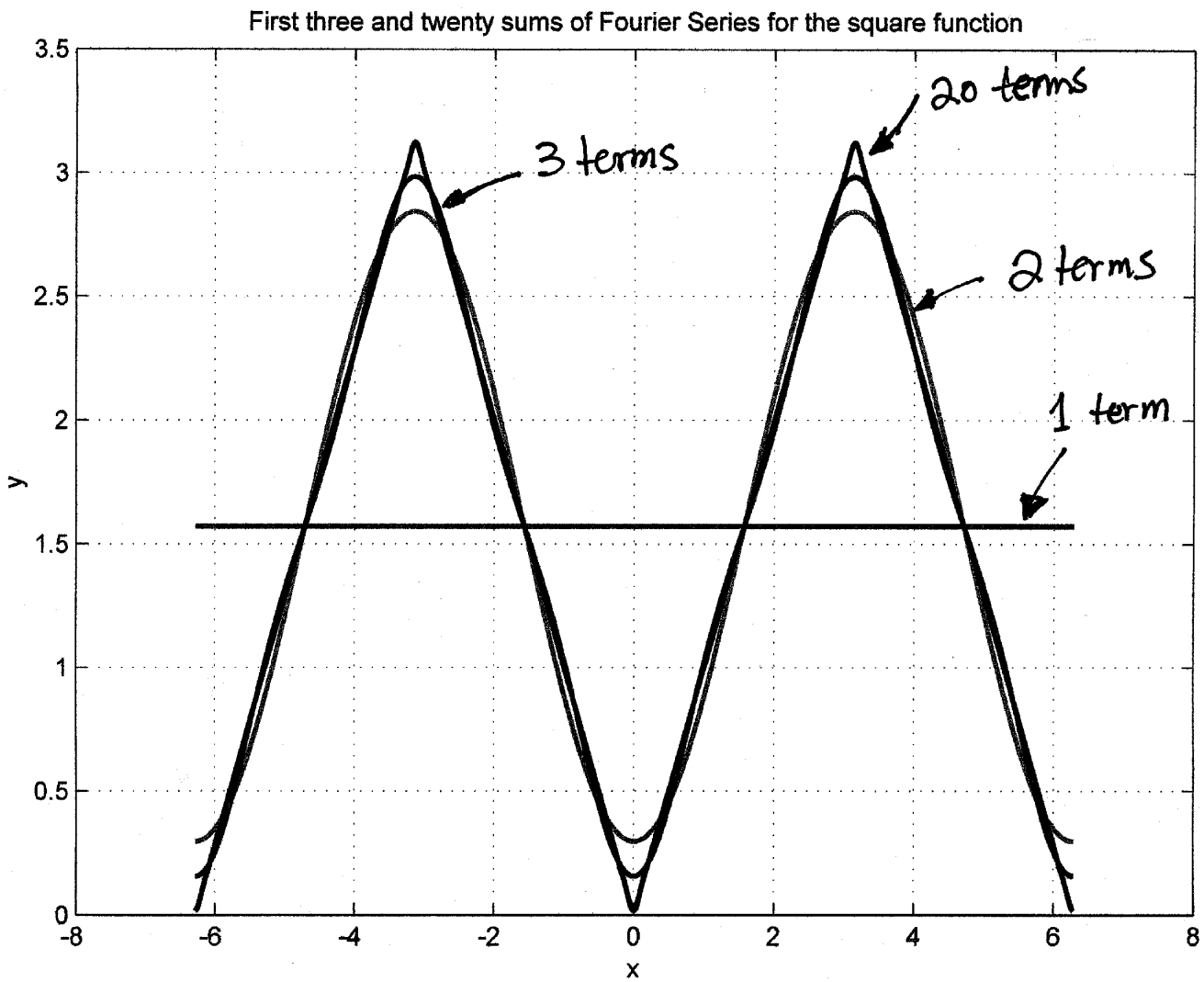
(2)



$$f(x) = |x| = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2}$$

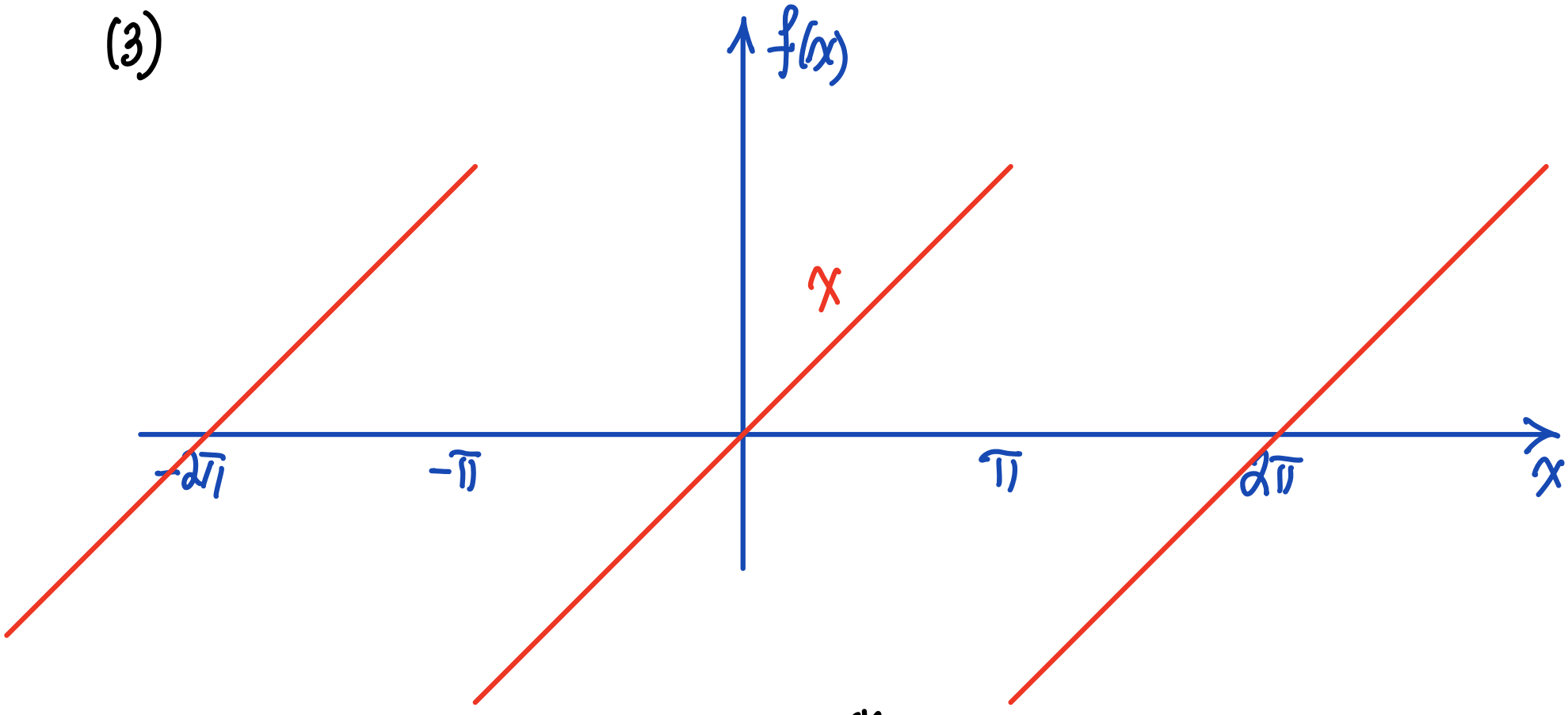
$(-\pi < x < \pi)$

p.26 (2)



Examples of Fourier Series

(3)



$$f(x) = x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \sin nx$$

$$(-\pi < x < \pi)$$

p.26 (1)

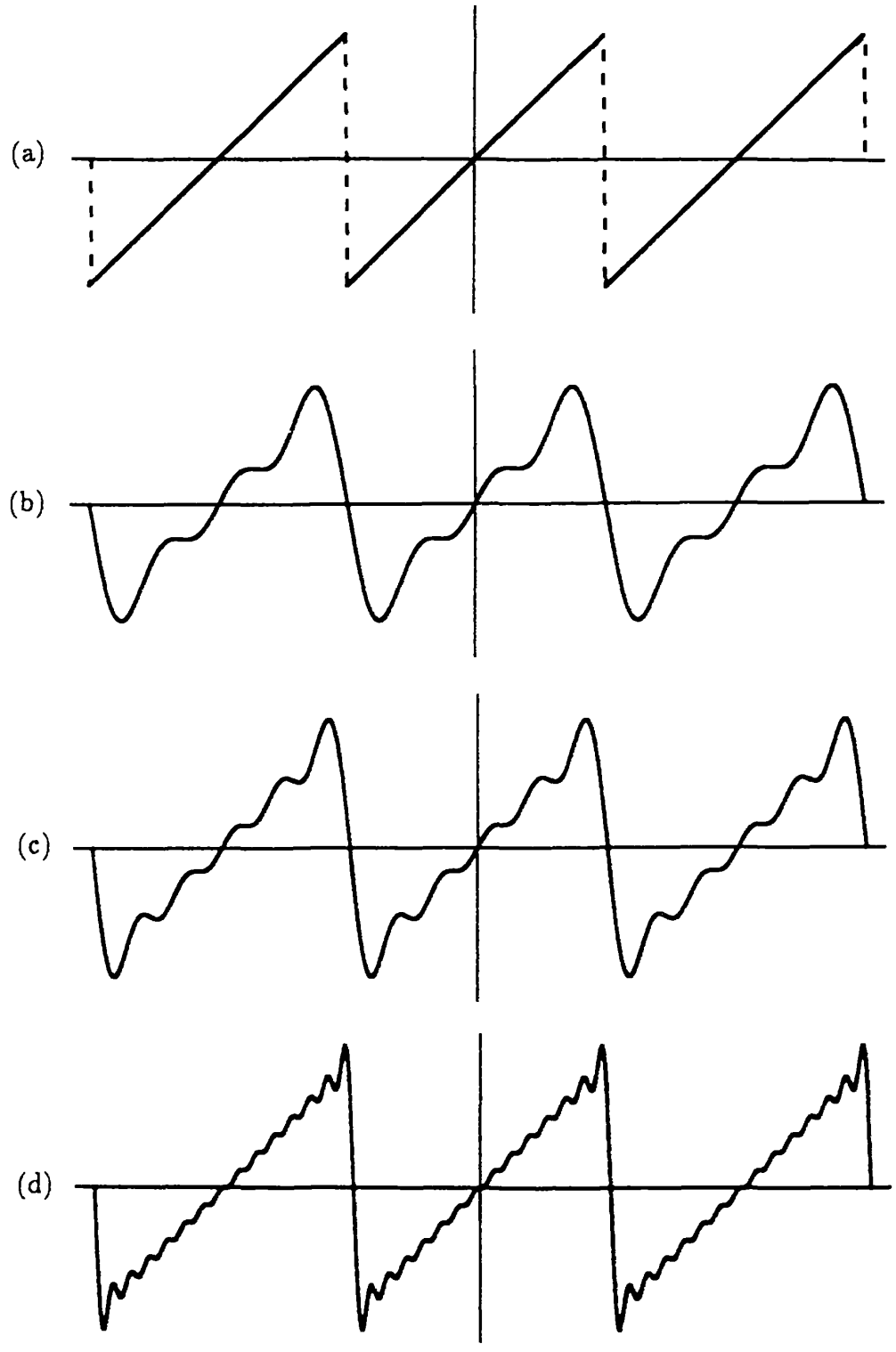
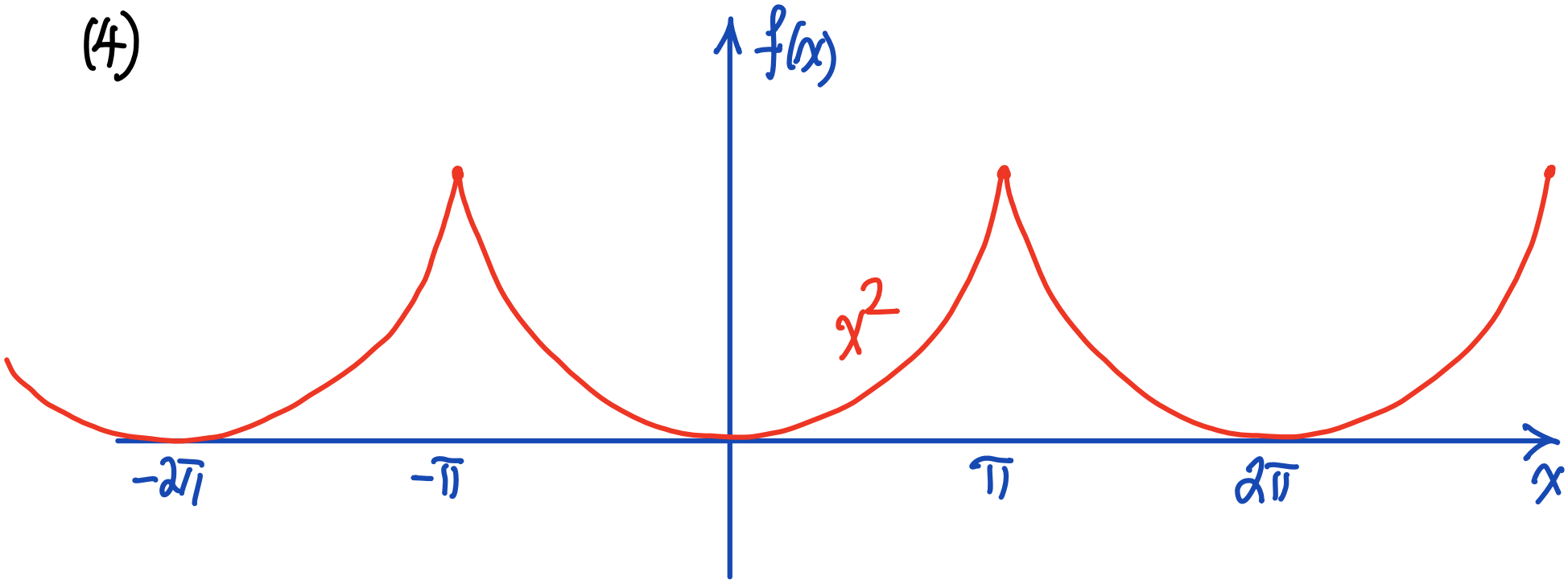


FIGURE 2.2. The sawtooth wave of Example 2 and some partial sums of its Fourier series: (a) the sawtooth wave, (b) S_3, (c) S_5, and (d) S_{14}, where $S_N = 2\sum_1^N (-1)^{n+1} n^{-1} \sin n\theta$.

Examples of Fourier Series

(4)



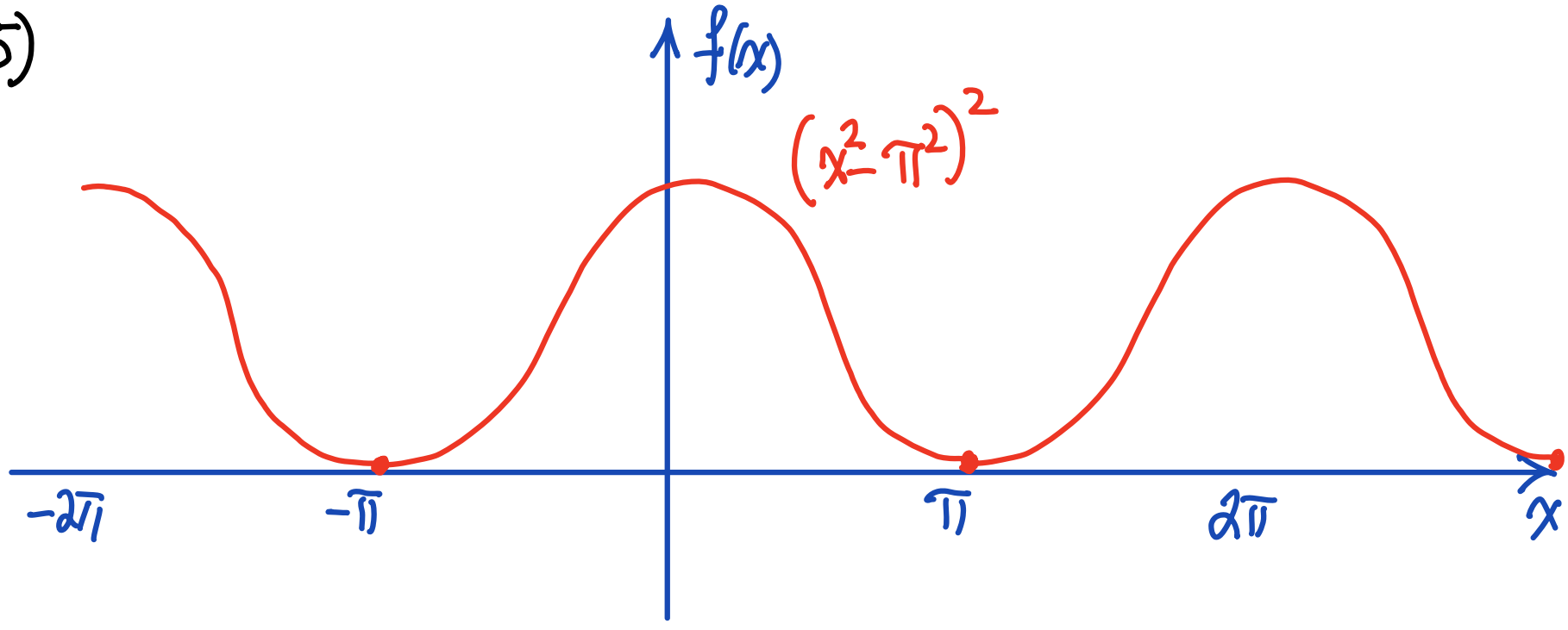
$$f(x) = x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

$$(-\pi < x < \pi)$$

p. 28 (16)

Examples of Fourier Series

(5)



$$f(x)=(x^2-\pi^2)^2=\frac{8\pi^4}{15}-48\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}\cos nx$$

$(-\pi < x < \pi)$