MA 520 Fourier Analysis and Boundary Value Problems
Spring 2024, Purdue University
(Instructor: Aaron N. K. Yip)
Practice problems on Fourier Transforms (No Need to Hand in)

Folland:
Section 7.1: #4, 5;
Section 7.2: #8, 12, 13;
Section 7.3: #1, 2, 4, 5.

Additional Problem.

1. Use Fourier Transform to solve the following equation:

$$\begin{aligned} u_t(x,t) &= Du_{xx} + cu_x(x,t) + hu(x,t), \ \ x \in (-\infty, \infty) \\ u(x,0) &= f(x) \end{aligned}$$

   (Note: It is possible to express the explicit solution using the classical heat kernel. Theorem 7.5(a) pp 214 might be useful.)

2. Using Fourier Transform to compute the following integral:

$$\int_{-\infty}^{\infty} x^n e^{-x^2}\, dx, \quad n = 0, 1, ...6.$$

   (Hint: Use Theorem 7.5(c) pp 214 and the fact that for any function $f(x)$, $\hat{f}(0) = \int_{-\infty}^{\infty} f(x)dx$.)