MA 543 Spring 2025 (Aaron N. K. Yip)
Homework 1, due on Thursday, Jan. 30th, in class

In the following, [M] refers to our official textbook by Meiss, *revised edition*, which is available online through the Purdue Library page.

As mentioned in the course policy, you can submit as a group consisting of up to three people. You are also allowed to consult and utilize online resouces and information, such as Wikipedia, plotting routines, and so forth. But submitting your solution as a complete "duplication" of online output is not acceptable. Your solution should explain your *solution and thought process in a clear and comprehensive way.*

1. [M Section 1.8]: #4, 6.

2. [M Section 2.9]: #2, 10(a,b,c), 15(a,b,c), 16, 19(c).

   For #2, the Hamiltonian system is given as: $\dot{x} = \partial_y H(x, y)$, and $\dot{y} = -\partial_x H(x, y)$.

3. Consider the following one-dimensional ODE:

$$\dot{x} = -x + x^2, \;\; x(0) = x_0, \;\; t > 0.$$

   (a) Solve the above system explicitly, in terms of the initial data $-\infty < x_0 < \infty$. (Hint: use the technique of separable equation.)

   (b) Determine/classify/characterize the behaviors of $x(t)$ as $t \longrightarrow +\infty$ in terms of x_0.

   (c) Illustrate your answers in a x-vs-t graph.

4. Given a matrix function $A(t)$, in class, we have introduced the concept of *fundamental matrix*, which solves the following matrix equation:

$$\frac{d}{dt}\Phi(t) = A(t)\Phi(t), \;\; \Phi(0) = I.$$

   It was also mentioned that $\Phi(t)^{-1}$ exists. You have the opportunity to prove this fact here using the following two methods.

   (a) Introduce the matrix function $\Psi(t)$ which solves

$$\frac{d}{dt}\Psi(t) = -\Psi(t)A(t), \;\; \Psi(0) = I.$$

   Assume the existence of $\Psi(t)$. Show that $\Psi(t)$ *is the inverse* of $\Phi(t)$, i.e. $\Psi(t)\Phi(t) = I$ by computing $\frac{d}{dt}\big(\Psi(t)\Phi(t)\big)$.

   (b) Make use of Abel formula, [M, Theorem 2.34].