MA 543 Spring 2025 (Aaron N. K. Yip)
Homework 3, due on Thursday, Feb. 27th, in class

In the following, [M] refers to our official textbook by Meiss, *revised edition, 2017*, which is available online through the Purdue Library page.

As mentioned in the course policy, you can submit as a group consisting of up to three people. You are also allowed to consult and utilize online resouces and information, such as Wikipedia, plotting routines, and so forth. But submitting your solution as a complete "duplication" of online output is not acceptable. Your solution should explain your *solution and thought process in a clear and comprehensive way.*

1. [M Section 4.13]: #1, 2(a,b), 8, 9, 10, 18.

2. Consider $\frac{d}{dt}X = F(X)$, $X(0) = X_0$. Even if the system is globally asymptotically stable, i.e. no matter what X_0 is, it holds that $X(t) \longrightarrow 0$ as $t \longrightarrow +\infty$, it does not mean that $\|X(t)\|$ will go to zero monotonically. In many (or almost all) cases, $\|X(t)\|$ will momentarily increase first before decay to zero.

   Consider the following 2×2 linear system:

   $$\frac{d}{dt}X = \begin{pmatrix} -1 & 10 \\ 0 & -2 \end{pmatrix} X, \quad X(0) = X_0.$$

   (Note that the above system is globally asymptotically stable because the matrix has only negative eigenvalues.)

   Suppose some engineering application mandates that you want $\|X(t)\|$ to be no more than 0.1, i.e. you want $\|X(t)\| \leq 0.1$, for all $t > 0$. How small should $\|X_0\|$ be? In other words, find $\delta > 0$ such that for any X_0 with $\|X_0\| \leq \delta$, you are guaranteed that $\|X(t)\| \leq 0.1$ for all $t > 0$. "Obviously", you would like to choose δ to be as big as possible.

   (In the above, $\|X\|$ refers to the usual Euclidean length of a vector in R^n.)